Özgür Kişisel

Toric Varieties I

(1)

General motivation

Algebraic variety is "the common zero locus of set of polynomials"
(usually we insist these are irreducible)

Classification up to iso is crazy hard

1-Dim	2-Dim
lines	waah!
elliptic curves	
⋮	

Morphisms — Two types

(1) Regular morphism: maps are polynom/rat'l funct defined everywhere on domain

(2) Rational morphism: maps are defined on an open subset of domain

Ex $x \mapsto (1/x, x)$

not def @ 0

Varieties / $\mathbb{C}$

Rational varieties: Varieties birational to $\mathbb{C}^n = \mathbb{A}^n$ ← Don't care about ∞

⇔ $V \to \mathbb{C}^n$ rational, $\mathbb{C}^n \to V$ rational } comp is rational id. (id on open set)

Ex $\mathbb{P}^n = \mathbb{A}^n \amalg \mathbb{P}^{n-1}$ is a rational variety.

Toric varieties: Rational varieties which contain an open torus $(\mathbb{C}^*)^n$ as a Zariski open set. so that $(\mathbb{C}^*)^n$ action extends over variety.

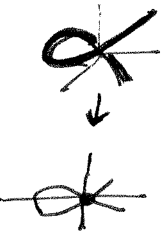
$(\mathbb{C}^*)^n = (\mathbb{C}\setminus\{0\})^n \supseteq (S^1)^{\times n}$

⇝ Like a compactification of an algebraic group....

Toric bundles: Vector bundes on toric varieties w/ compatible torus action.

(2) Why are toric varieties useful?

- They generalize $\mathbb{A}^n$, $\mathbb{P}^n$
- Natural to look at when resolving singularities EX

  $y^2 = x^2(x+1)$

  → Blow up at point cuts out point ($\sim \mathbb{C}^*$) sticks in $\mathbb{P}^1$
- Testing ground for conjectures
  - EX: Conjectures about mirror-symmetry
  - EX: Vector bundles problems (more on this later)
  - EX: Derived algebraic geometry
- Has been useful for problems in representation theory:

  → Klyachko: ~~A~~ solution of ~~Alex~~ Horn's conjecture

  What do you know about eigenvalues of $(A+B)$ if you know eigenvalues of A and B?

  → Horn conj big list of inequalities was complete list.

  EX sum eigenval A + sum eigenval B = sum eigenval $(A+B)$

  ⋮

→ Next week: Explicit definition
Explanation of diagrams (pictures)
Explanation of some work of Özgür.