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"Valued Fields"

Def: Valued Field is (K, Γ, k, v)

K = field

Γ = value group (ordered)

k = residue field $\mathcal{O}_v / \mathcal{M}_v$

$v: K^\times \longrightarrow \Gamma$ valuation

$v(x+y) \geq \min(v(x), v(y))$

$v(xy) = v(x) + v(y)$, $v(0) = \infty$

w/ valuation ring $\mathcal{O}_v = \{a \in K \text{ w/ } v(a) \geq 0\}$

max ideal $\mathcal{M}_v = \{a \in K \text{ w/ } v(a) > 0\}$

Def: Regular group has all open intervals w/ n elements contains an element ng ← "an n-divisible element"

(**Lemma:** Regular (ℤ-group?) ⟺ $G/_{nG} \cong \{0, \ldots, n-1\}$ isom. of ordered groups (??))

Def: A ℤ-group is a regular group that is discrete ↳ has a smallest elmt

→ All ℤ-groups are elem. equiv to ℤ.

Def: An ordered group is dense if for $\alpha < \beta$ there is $\alpha < \gamma < \beta$

FACT:
- All regular groups are either < ℤ-group / dense group
- An ordered group is regular if Γ/Δ is divisible ∀ nonzero convex subgroup Δ of Γ.

Def: A valued field K is extremal if ∀ polynomial $P(x_1, \ldots, x_n)$ / K the set

$\{v(P(a_1, \ldots, a_n)) \text{ w/ } a_1, \ldots, a_n \in \mathcal{O}_v\} \subseteq \Gamma_\infty$

has a max element.

Recall: $v(0) = \infty$

(2)

$\boxed{\text{Thm: } (K, \Gamma, k, v) \text{, Henselian valued field w/ } \Gamma \text{ a } \mathbb{Z}\text{-group and char } k = \text{char } K = 0. \text{ Then } K \text{ is extremal.}}$

$\bar{f}(\bar{c}) = 0$ over residue field

$\bar{f}'(\bar{a}) \neq 0$ over res. field

Recall:

Henselian means $\forall$ f, a / $\mathcal{O}_v$ if $v(f(a)) > 0$ & $v(f'(a)) = 0$
then there is ε w/ $f(a+\varepsilon) = 0$.

→ Equivalent ~~to~~ to extremal restricted ~~to~~ to polynomials of one variable.

Let $A \subseteq B$ be two structures of a first order language

Def: A is existentially closed ~~in~~ in B if every existential sentence w/ parameters in A that is true in B is also true in A.