



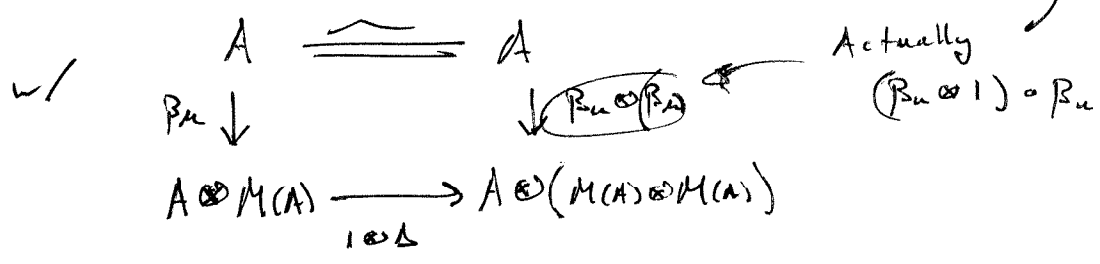
Remark: If  $(M(A), \beta_u)$  exists, then

- $M(A)$  is a bialgebra
- $\beta_u$  makes  $A$  an  $M(A)$  comodule algebra  
( $\beta_u$  gives coaction)
- $M(A)$  is the universal bialgebra w/ coaction

$\leadsto$  This is the direct analog of automorphism group.

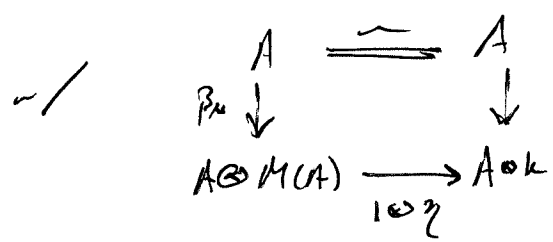
①  $M(A)$  is a bialgebra b/c  $(M(A) \otimes M(A), \beta_u \otimes \beta_u)$  is comod.

universality  $\Rightarrow \exists \Delta: M(A) \rightarrow M(A) \otimes M(A)$



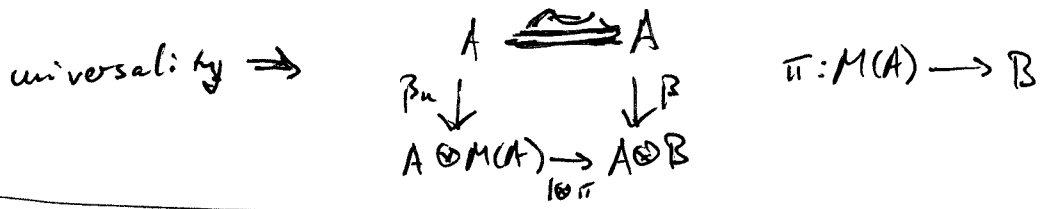
unit comes from comod. ( $k, \text{id} \otimes 1$ )

$\Rightarrow \exists \eta: M(A) \rightarrow k$



HW: These satisfy bialgebra axioms

③ If  $\beta: A \rightarrow A \otimes B$  is another bialg w/ coaction



HW: compatible w/ bialgebra coactions

Existence of  $M(A)$

→ Let  $A$  be finite dimensional.

Plan: Construct  $M(A)$  in the case where  $A$  not necess. unital  
 - deal w/ unit later. (formally adjoin unit)

(In this case comultiplying means only  $A \xrightarrow{\beta} A \otimes B$  multiplicative)

Fix a basis  $\{e_i\}_0^{\dim(A)-1}$  of  $A$ .

• define  $M_1(A) = \mathcal{K} \langle \underbrace{t_j^i}_n \rangle$  where  $i, j \in \{0, \dots, \dim(A)-1\}$

where  $c_{ij}^a t_a^k \sim c_{ab}^k t_i^a t_j^b$   $\left\{ \begin{array}{l} c_{ij}^k \text{ are structure constants of } \{e_i\} \\ e_i \cdot e_j = c_{ij}^k e_k \end{array} \right.$

coalg.  $\left( \begin{array}{l} \Delta(t_j^i) = t_a^i \otimes t_j^a \\ \eta(t_j^i) = \delta_j^i \end{array} \right. \leftarrow \text{(Matrix multiplication.)}$

comas.  $(\beta_u(e_i) = e_a \otimes t_i^a)$

→ Check that  $\beta_u$  is multiplicative:

$$\begin{aligned}
 \beta_u(e_i \cdot e_j) &= \beta_u(c_{ij}^k e_k) = c_{ij}^k \beta_u(e_k) \\
 &= c_{ij}^k (e_k \otimes t_a^k) \\
 &= c_{ab}^k e_k \otimes t_i^a t_j^b \\
 &= e_a e_b \otimes t_i^a t_j^b \\
 &= \beta_u(e_i) \cdot \beta_u(e_j)
 \end{aligned}$$

} swap notation  $a \leftrightarrow k$ .

$\leadsto$  check universality:

If  $(\beta, \beta)$  is comas.

$$\text{say } \beta(e_i) = e_a \otimes \overline{t_i^a}$$

$\beta$  mult  $\implies$

$$c_{ij}^a \overline{t_i^h} = c_{ab}^k \overline{t_i^a} \overline{t_j^b}$$

Define:  $\pi: M_1(A) \rightarrow B$  by

$$e_j^i \mapsto \overline{t_j^i}$$


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