RESEARCH STATEMENT

SALIH AZGIN

My area of expertise is model theory which is a branch of mathematical logic. In my research I focus on model theoretic properties of algebraic structures. This involves focusing on certain properties of these structures forgetting about the rest. So model theoretic tools can be used to investigate algebraic structures.

My research program focuses on model theoretic and algebraic properties of valued fields. Hasse's substantial contributions to the arithmetic of fields using ideas from valuation theory drew attention to the subject. Later valuation theory found applications in algebraic geometry and developed into a vast research area that has strong ties with analysis, algebra, geometry, number theory and model theory.

It was the work of Ax, Kochen and Ershov ([1], [6]) that popularized valued fields in model theory. Through the model theoretic properties of certain valued fields Ax, Kochen proved an asymptotic version of the following conjecture of Artin: Every homogeneous polynomial of degree d over $\mathbb{Q}_p$ in d^2+1 variables has a nontrivial zero in $\mathbb{Q}_p$[1]. This result has attracted researchers to the subject and today model theory of valued fields constitutes an essential part of valuation theory and its applications.

In my current and future work I aim to achieve broader expertise in both model theory and algebra as well as advancing my contributions to the theory of valued fields. Below is a summary of my research program, including accomplishments so far, current projects and future plans.

Valued Difference Fields. *Valued difference fields* are valued fields equipped with a distinguished automorphism. The study of valued difference fields varies greatly depending on the interaction between the valuation and the distinguished automorphism. One natural and interesting example of valued difference fields is the field of Witt vectors over the algebraic closure of $\mathbb{F}_p$ equipped with the lifting of the Frobenius automorphism. In this structure the valuation of every element is preserved by the automorphism. The model theoretic properties of this structure have been studied in [2]. In joint work with Lou van den Dries (based on my PhD thesis), we generalize some of the results of [2]. This article is accepted, pending revisions, by the Journal of the Institute of Mathematics of Jussieu.

Another article, to appear in the Journal of Algebra, based on my PhD thesis considers model theoretic and algebraic properties of valued difference

[1] This conjecture is false.

fields where the automorphism increases the valuation of elements from the maximal ideal. These structures, even in characteristic zero, pose difficulties that resemble the problems that arise in positive characteristic valued fields. The elementary theory of positive characteristic valued fields in general is not known. Consequently, there are still important open problems in this research topic. One interesting valued difference field is the field of logarithmic-exponential series equipped with the automorphism that sends a formal series $f(t)$ to $f(e^t)$, see [3]. The elementary theory of this structure is not known. I have some preliminary results and a promising program for this problem. In collaboration with Franz-Viktor Kuhlmann, we try to apply the techniques he invented for positive characteristic valued fields, see [9], to valued difference fields. This is a long term project that may produce high impact applications including decidability/undecidability results for fragments of the elementary theory of $(\mathbb{R}, \exp)$.

Extremal Valued Fields. In collaboration with Franz-Viktor Kuhlmann and Florian Pop we classify extremal valued fields, article submitted to the Proceedings of the AMS in July 2009. A valued field is *extremal* if values of every polynomial in several variables reach a maximal valuation when evaluated at tuples from the valuation ring[2]. This notion was introduced by Ershov, see [5], but we show that his original definition is flawed. We fix this flaw and prove a surprising classification theorem for extremal valued fields. Henselian valued fields with residue characteristic zero were expected to be extremal but this is far from being the case. There are seemingly stronger notions similar to extremality that arise from considering the valuations of polynomials evaluated at tuples from certain definable sets in the field. I plan to investigate these notions and their relation with extremality. The motivation for these notions comes from trying to axiomatize the elementary theory of $\mathbb{F}_p((t))$ which is one of the most important open problems in the area.

Reverse Quantifier Elimination. Many algebraic structures are studied via quantifier elimination (in a suitable language) in model theory. The question is raised the other way around in [4] to obtain: If a field has quantifier elimination in the language of rings then it is algebraically closed. If a field has quantifier elimination in the language of ordered rings then it is real closed. Quantifier elimination in valued fields is a complicated issue and there are many languages available. In [4] and recently in [10] certain valued fields that admit (relative) quantifier elimination are proved to be henselian. However there is still much room for improvement in those results. Using the relative quantifier elimination in the Denef-Pas language for valued fields, I am working on generalizing the results of [10] which assume that the value group has rank 1, to finite rank value groups.

[2]A similar assertion for the Laurent series field $\mathbb{R}((t))$ is known as Artin approximation.

Further Projects. One can define exponentiation in $\mathbb{Q}_p$ as in the reals. Even though the p-adic exponentiation is not defined on the entire field of p-adics it possesses many of the properties of the real exponentiation. What can be said about the model theoretic properties of $\mathbb{Q}_p$ equipped with the p-adic exponentiation? It would be too optimistic to expect to fully understand the elementary theory of this structure since the p-adic Schanuel's conjecture can be formulated in it. However, it is conceivable that results like model completeness are within reach given all the recent developments in valued fields with added structure like valued difference fields and valued differential fields.

References

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