Research Interests

I am particularly attracted to problems in mathematics requiring the simultaneous use of ideas from distant fields. I have conducted at least one bit of research in the fields of Algebraic geometry, mathematical physics, combinatorics, differential geometry, number theory, and the theory of mechanisms. More precisely, in each of these general areas, I have worked on problems that could be listed under the following (still rather broad) titles:

- Algebraic geometry: Relations between moduli problems and completely integrable systems, vector bundles (especially on toric varieties), applying tropical ideas to problems about hyperplane arrangements, and Hurwitz schemes in relation to moduli spaces of certain surfaces.
- Mathematical physics: The algebraic aspects of completely integrable systems, positive mass type problems in general relativity, and the quantum Hall effect (especially concerning its relation to noncommutative geometry).
- Combinatorics: Tiling problems.
- Differential geometry: Geometric flows.
- Number theory: Some products which are never squares.
- The theory of mechanisms: Deployable polyhedra.

Below, I list my published work concerning these interests. In the final section, I listed the projects that I am either currently working on or those that I have an intention to give a try to in the near future. Each of the titles is followed by a synopsis (not coinciding with the formal abstract). These indicate, in addition to the results in the paper, some ideas about possible follow-ups. I have not listed some unpublished work which relate to some of the titles above, especially if I am not likely to work on them in the new future. The published items are listed in reverse chronological order.

1. Past research:

1.1. **(with E. Gürel)** ***On the products*** $(1^\mu+1)(2^\mu+1)\dots(n^\mu+1)$ **(accepted for publication in *Journal of Number Theory*).** We prove that the product $(1^3+1)(2^3+1)\dots(n^3+1)$ is never squarefull, and in particular never a square. J. Cilleruelo had previously proven that $(1^2+1)(2^2+1)\dots(n^2+1)$ is a square only for $n=3$, and we essentially show that the same technique can be employed for the cubic case. One uses an effective estimate to handle sufficiently large values of n, and an elementary argument for the smaller values. It is quite critical what the precise estimates are, otherwise the second part of the proof becomes intractable. We are trying to find similar results for products of the type $(1^2+d)(2^2+d)\dots(n^2+d)$ for any positive square free d, and hope to prove theorems of the form "for d less than, say, 1000000, below is the list of all n (we know that there are finitely many of them) for which the product is a square". We need to prove some effective character-sum estimates first in order to make this work.

1.2. **(with Ö. Sarıoğlu and B. Tekin)** ***Cotton flow, published in*** **Classical and Quantum Gravity (25), 2008.** As is well known, the Ricci flow and its variants have been tremendously effective in tackling long standing problems in 3-manifold topology. What we suggest in this article is the possible exploitation of another geometric flow, which we call the Cotton flow. The Cotton tensor plays a role analogous to the Weyl tensor in higher dimensions (which identically vanishes in 3-dimensions), in that, its vanishing is equivalent to the conformal flatness of the metric. We study the evolution of homogenous metrics in 3-dimensions in this article. These metrics are rather special, therefore the analysis is relatively easy. On the other hand, by geometrization, we know that they are ubiquitous. This work could be furthered either in the direction of looking for new pathways or shortcuts to the proof of the geometrization conjecture, or to tackle other geometric problems such as topologically classifying all 3-manifolds which admit a metric conformal to a flat metric.

1.3. **(with Ö. Öztürk)** ***Toric varieties and the diagonal property,*** **accepted for publication in** ***Arrangements, Local Systems and Singularities, Lecture notes of the CIMPA 2007 summer school,*** **Birkhauser, Prog. in Math. series (to appear in 2009).** We say that a smooth algebraic variety X of dimension n over an algebraically closed field satisfies the diagonal property if there exists a vector bundle of rank n on $X \times X$ such that it has a section whose zero scheme coincides with the diagonal in $X \times X$. This is a property of obvious geometric significance. The general question is "Which varieties possess the diagonal property?", and to my knowledge, this was posed first by Fulton and Pragacz. In [1] Pragacz, Srinivas and Pati obtained some detailed results in the case where X is a surface, and some partial results for higher dimensions. On surfaces, one can take advantage of Serre's construction to prove existence in many cases in which the answer is positive, but this idea doesn't work in the same way in ranks higher than 2. Pragacz then suggested us to look at the case of toric varieties, since there the combinatorial description may give additional insight to what is happening. We started from the case of toric surfaces; in this case the diagonal property is satisfied, and this had already been shown in [1], however we construct the bundle using the fan data alone. This shows additionally that the bundle can be chosen to be equivariant under the action of a two dimensional torus. This partly expository article is our first publication in this direction, giving details of the construction only for three examples of toric surfaces. A systematic treatment for all toric surfaces will follow, and we are working on the case of higher dimensional toric varieties, where we suspect that the answer is not always positive. This might lead to a first example of a rational variety that does not satisfy the diagonal property.

1.4. **(with G. Kiper and E. Söylemez)** ***A family of deployable polygons and polyhedra,*** **Mechanism and Machine Theory (43) no. 5, 2008, pp 627–640.** In mechanism theory, it is of interest to design structures which can be expanded or collapsed using a single degree of freedom of motion. Polyhedral mechanisms are quite popular in this area; one takes advantage of the symmetries for the movability of the unavoidably overconstrained design. Some applications are already present, such as in stadium roofs, or in the toy industry. An abstract study of deployable polyhedra is of interest, and there remain several open theoretical questions. In this article, a method for systematically designing classes of deployable polyhedra using a certain basic linkage element is introduced. My role in this article has been minor compared to the other two authors.

1.5. ***On quadratic Poisson brackets,*** **Journal of Mathematical Physics (46) no. 4 , 2005.** In this paper, I explain how some relatively complicated quadratic Poisson brackets can be obtained from simpler brackets in a natural manner. The underlying idea is to use matrix factorizations. I show that both some r-matrix brackets, and some others can be obtained in this manner. I then use this method to construct a quadratic Poisson bracket for a discrete space-continuous time version of the KP hierarchy that is periodic in the space dimensions. Using these brackets, one can prove the complete integrability of the system, however I do not yet know a general framework on how conserved quantities fit into this construction. I think that a more invariant way of making this construction will coincide with a certain generalization of the Hitchin system.

1.6. ***Integrable systems and Gromov-Witten theory,*** **Topics in Cohomological Studies of Algebraic Varieties Birkhauser, Trends in Mathematics, 2005, pp 135–161.** This is an expository article intended to explain some basic constructions in the algebraic theory of completely integrable systems, how they relate to Gromov-Witten theory, and highlight some of the main ideas appearing in Kontsevich's proof of Witten's conjecture. It is intended for the reader who is just starting to look at the topic and who wants to get an outline of the horde of ideas used in the proof without getting lost, on the other hand almost none of the technical tools are presented rigorously.

1.7. ***Polyomino Convolutions and Tiling Problems* Journal of Combinatorial Theory, Series A (95), 2001, pp 373–380.** It is a classical question whether using copies of a given plane figure one can tile the whole plane, that is, cover it without any overlaps of positive measure. Let us specialize this problem to polyominoes. A polyomino is a plane figure made up of unit squares whose corners have integer coordinates. Even then, it is known that the problem is undecidable, that is, there is no algorithm that can certify whether an arbitrarily given polyomino can tile the plane or not. Many necessary conditions are known, most of which rely on various coloring arguments. In this article, I introduce a new necessary condition based on a convolution operation of polyominoes modulo a given integer, and show that several classes of polyominoes cannot tile the plane. I also give an argument to show that the method detects some cases inaccessible by any generalized coloring argument, therefore proving its authenticity.

2. Now or the Near Future:

2.1. **(with E. Özkan) *Nilpotent group actions on the moduli spaces of vector bundles on* $\mathbb{P}^2$.** The moduli spaces of stable vector bundles of a given rank on $\mathbb{P}^2$ are intriguing objects. There has been much work done towards understanding their topological properties. As in many other situations, a popular way to uncover such properties is to understand the fixed point data under a torus action on this space. We note in particular the contributions of Klyachko, who showed that the category of equivariant bundles is equivalent to a category of certain filtrations of a vector space. Our aim is to study the fixed schemes of a nilpotent vector field acting on these moduli spaces (these exist, which can easily be seen by considering such actions on the base). When this idea is applied to flag varieties instead, nilpotent vector fields give new information about the topology. We hope to obtain analogous information here.

2.2. **(with H. Güntürkün) *Tropical hyperplane arrangements.*** There are many long-standing open problems about hyperplane arrangements. These range from the existence of arrangements having certain numerical properties to questions about the topology of the complements of the arrangements. The idea of looking at tropical limits has been very useful in the areas of real algebraic curves and the enumerative geometry of curves. We are trying to see if there are some applications to hyperplane arrangements. The arrangement itself is a combinatorial model, however it is too complicated in general. Certain simpler associated combinatorial data codifying incidence, such as orthogonal Latin squares, can be too simple to detect the differences required to solve the problems. We think that tropical limits stand somewhere in between.

2.3. **(with B. Tekin)** ***A relativistic version of the Quantum Hall Effect and noncommutative geometry.*** The ideas here are in very primitive form yet. In a nutshell, we want to adapt J. Bellisard's method for proving the integrality of certain quantities in the Quantum Hall Effect using noncommutative geometry to the relativistic case. The aim is to explain some recent experimental observations whose underlying models effectively coincide with relativistic QHE.

2.4. **(with M. Çelik)** ***A method of construction of R-matrices.*** Our idea is to construct quantum R-matrices using matrix factorizations, motivated by the construction of Quadratic Poisson Brackets described above. This project is also in its infancy stage.

References

[1] Pragacz, P. , Srinivas, V. , Pati, V. *Diagonal subschemes and vector bundles*, Pure Appl. Math. Q. 4 (2008), no. 4, part 1, 1233–1278.