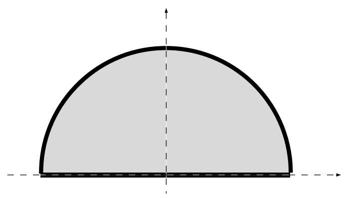
M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 2A	
Code : *Math 120* Acad.Year : *2013-2014* Semester : *Spring* Date : *14.5.2014* Time : *18:40* Duration : *25 minutes*	Last Name: Name: KEY Signature:	Student No: 666
	2 QUESTIONS ON 2 PAGES TOTAL 20 + 2 BONUS POINTS	

1	2			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (8 pts.) Let R be the upper half disk bounded by the semi-circle $\{x^2+y^2=4\,,\ y\geq 0\}$ and the line segment $\{y=0\,,\ -2\leq x\leq 2\}$, oriented counter-clockwise. Call this boundary curve C. Evaluate the following line integral **using Green's Theorem**.



$$\oint_C \langle \arctan\left(\ln(x^2+1)\right) - y^3, x^3+54x\rangle \bullet d\mathbf{r}$$

$$= \iint_D (3x^2+3y^2+54)\, dA$$

$$= \int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=2} 3r^2 r\, dr\, d\theta \quad + 54\iint_D dA$$

$$= \int_0^{\pi} 3\cdot \frac{r^4}{4}\Big|_0^2 \quad + 54\ \text{Area}(D)$$

$$= 12\pi \quad + 54\cdot\frac{\pi\cdot 2^2}{2}$$

$$= 12\pi \quad + 108\pi$$

$$= 120\pi$$

2. $(4+4+6=14$ pts.) Represent the following functions as a power series about the point $c=0$, that is, in powers of x.

Do not forget to find the domains on which the formulas are valid.

(a) $\dfrac{1}{x-2} = \dfrac{-1}{2-x} = \dfrac{-1}{2}\dfrac{1}{1-x/2}$

$$= \frac{-1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n \quad ; \left|\frac{x}{2}\right|<1$$

$$= -\sum_{n=0}^{\infty}\frac{x^n}{2^{n+1}} \quad ; |x|<2$$

(b) $\dfrac{1}{2x-1} = \dfrac{-1}{1-2x}$

$$= -\sum_{n=0}^{\infty}(2x)^n \quad ; |2x|<1$$

$$= -\sum_{n=0}^{\infty}2^n x^n \quad ; |x|<\frac{1}{2}$$

(c) $\dfrac{5x-4}{(2x-1)(x-2)}$ *Hint:* Use Partial Fraction Decomposition

$$\frac{5x-4}{(2x-1)(x-2)} = \frac{A}{2x-1}+\frac{B}{x-2} = \frac{A(x-2)+B(2x-1)}{(2x-1)(x-2)} = \frac{(A+2B)x+(-2A-B)}{(2x-1)(x-2)}$$

$$\Longrightarrow A=1, \quad B=2$$

$$\frac{5x-4}{(2x-1)(x-2)} = \frac{1}{2x-1}+\frac{2}{x-2}$$

$$= -\sum_{n=0}^{\infty}2^n x^n + 2\left(-\sum_{n=0}^{\infty}\frac{x^n}{2^{n+1}}\right) \quad ; |x|<2 \ \& \ |x|<\frac{1}{2}$$

$$= \sum_{n=0}^{\infty}-2^n x^n - \sum_{n=0}^{\infty}\frac{x^n}{2^n} \quad ; |x|<\frac{1}{2}$$

$$= \sum_{n=0}^{\infty}\left[-2^n-\frac{1}{2^n}\right]x^n \quad ; |x|<\frac{1}{2}$$