# M E T U
# Northern Cyprus Campus

| | Calculus With Analytic Geometry<br>Short Exam 1 | |
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| Code : *Math 119*<br>Acad.Year: *2011-2012*<br>Semester : *Summer*<br>Date : *06.7.2012*<br>Time : *16:40*<br>Duration : *30 minutes* | Last Name:<br>Department:<br>Section:<br>Recitation: | Name:<br>Student No:<br>Signature: |
| | 2 QUESTIONS ON 2 PAGES<br>TOTAL 50 POINTS | |

| 1 | 2 | | | |
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**Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!**

**Please do not write on your desk!**

1. ($8 \times 5 = 40$ pts.) Evaluate the limit, if it exists.

(a) $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2+4x+4}$ $= \dfrac{4-4}{16} = 0$

$\lim\limits_{x\to 2} x^2+4x+1 = 4+8+4 = 16$

$\boxed{0}$

(b) $\lim\limits_{x\to -2} \dfrac{x^2+3x+2}{x^3+8}$ $= \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x^2-2x+4)}$

$= \dfrac{-2+1}{4+4+4} = \dfrac{-1}{12}$

$\boxed{-1/12}$

(c) $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{x-1}$ $\cdot \dfrac{\sqrt[3]{x}^2+\sqrt[3]{x}+1}{\sqrt[3]{x}^2+\sqrt[3]{x}+1} = \lim\limits_{x\to 1} \dfrac{\cancel{x-1}}{\cancel{(x-1)}\cdot(\sqrt[3]{x}^2+\sqrt[3]{x}+1)}$

$= \dfrac{1}{1+1+1} = \dfrac{1}{3}$

$\boxed{1/3}$

(d) $\lim\limits_{x\to -3} \dfrac{x^3-1}{(x-5)^2(x+3)}$ dne

$-3 \quad 1 \quad 5$

$+ \;\phi\; - \;\phi\; + \;8\; +$

$\lim\limits_{x\to -3^-} \dfrac{x^3-1}{(x-5)^2(x+3)} = \infty$ $\quad \lim\limits_{x\to -3^+} \dfrac{x^3-1}{(x-5)^2(x+3)} = -\infty$

$\boxed{\text{dne}}$

(e) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-x^2}{1-\sqrt{x}}$ $\cdot \dfrac{\sqrt{x}+x^2}{\sqrt{x}+x^2} \cdot \dfrac{1-\sqrt{x}}{1-\sqrt{x}} = \lim\limits_{x\to 1} \dfrac{(x-x^4)(1+\sqrt{x})}{(1-x)(\sqrt{x}+x^2)}$

$= \lim\limits_{x\to 1} \dfrac{x\cancel{(1-x)}(1+x+x^2)(1+\sqrt{x})}{\cancel{(1-x)}(\sqrt{x}+x^2)} = \dfrac{1.3.\cancel{2}}{\cancel{2}} = 3$

$\boxed{3}$

(f) $\lim_{x\to 5} \frac{|x-5|}{x^2-5x}$

$$|x-5| = \begin{cases} x-5 & x \geq 5 \\ -(x-5) & x < 5 \end{cases}$$

$$\lim_{x\to 5^+} \frac{|x-5|}{x^2-5x} = \lim_{x\to 5^+} \frac{\cancel{x-5}}{x\cancel{(x-5)}} = \frac{1}{5}$$

$$\lim_{x\to 5^-} \frac{|x-5|}{x^2-5x} = \lim_{x\to 5^-} \frac{-\cancel{(x-5)}}{x\cancel{(x-5)}} = -\frac{1}{5}$$

$\boxed{\text{dne}}$

(g) $\lim_{x\to 0} x\cos\frac{1}{x}$ $= 0$ by squeeze thm

$$-1 \leq \cos\frac{1}{x} \leq 1 \Rightarrow \underset{\downarrow \atop 0}{-x} \leq x\cos\frac{1}{x} \leq \underset{\downarrow \atop 0}{x}$$

$\boxed{0}$

(h) $\lim_{x\to -1} \frac{\sin(x-1)}{x^2-2x+1}$ $= \frac{\sin(-2)}{4}$

$$\lim_{x\to -1} x^2-2x+1 = +1+2+1 = 4$$

$\boxed{\frac{\sin(-2)}{4}}$

2. (10 pts.) Using the definition of the limit, prove that $\lim_{x\to -1} 3x+2 = -1$.

<u>Aim</u>: For all $\varepsilon > 0$, find $\delta > 0$ s.t. if $0 < |x+1| < \delta$ then $|3x+2+1| < \varepsilon$.

$$|3x+2+1| = |3x+3| = 3|x+1| < 3\delta$$

Let $3\delta = \varepsilon \Rightarrow \delta = \varepsilon/3$

$\therefore$ Given $\varepsilon > 0$ choose $\delta = \varepsilon/3$. Then if $0 < |x+1| < \delta$ then

$$|3x+2+1| = 3|x+1| < 3\delta = 3\frac{\varepsilon}{3} = \varepsilon.$$

So $\lim_{x\to -1} 3x+2 = -1$