M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 3	
Code : *Math 120* Acad.Year: *2012-2013* Semester : *Summer* Date : *03.8.2013* Time : *15:45* Duration : *45 minutes*	Last Name: Name: Student No: Signature:	
	3 QUESTIONS ON 2 PAGES TOTAL 20 POINTS	

1	2	3			
			KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ **around your answers!**
Please do not write on your desk!

1. $(1 + 4 + 1 = 6$ pts.) Let $F(x, y) = \left\langle \frac{2x}{x^2+y^2} + 4x, \frac{2y}{x^2+y^2} \right\rangle$.

   (a) Show that $F(x, y)$ is a conservative vector field.

$$\frac{\partial}{\partial y}\left(\frac{2x}{x^2+y^2} + 4x\right) = \frac{-2x}{(x^2+y^2)^2} \cdot 2y + 0 = \frac{\partial}{\partial x}\left(\frac{2y}{(x^2+y^2)}\right)$$

   (b) Find a potential function $f(x, y)$ for $F(x, y)$, i.e., a function that satisfies

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial x} \right\rangle = \left\langle \frac{2x}{x^2+y^2} + 4x, \frac{2y}{x^2+y^2} \right\rangle$$

$$f = \int \frac{2y}{x^2+y^2}\, dy \underset{\substack{u = x^2+y^2 \\ du = 2y\, dy}}{=} \int \frac{du}{u} = \ln|u| + C(x) = \ln(x^2+y^2) + C(x)$$

$$\frac{\partial f}{\partial x} = \frac{2x}{x^2+y^2} + 4x \Rightarrow C'(x) = 4x \Rightarrow C(x) = 2x^2 + K$$

$$\boxed{f(x,y) = \ln(x^2+y^2) + 2x^2 + K}$$

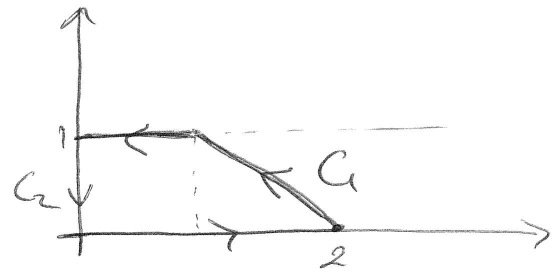
   (c) Evaluate the line integral $\int_L F(x, y) \bullet dr$ on the line segment L joining the point $(2, -1)$ to the point $(1, 2)$ **by using Fundamental Theorem of Line Integrals only.** Other methods will not receive any credit.

$$\int_L F \cdot dr = f(1,2) - f(2,-1) = (\ln(5) + 2 + K) - (\ln(5) + 8 + K)$$

$$= \boxed{-6}$$

2. $(1+1+2+4=8$ pts.) Let R be the region in the first quadrant bounded by the lines $y=1$ and $x+y=2$. Let C_1 be the oriented curve on the boundary of R, consisting of the line segments that go from $(0,0)$ to $(2,0)$ to $(1,1)$ to $(0,1)$. Let C_2 be the line segment from $(0,1)$ to (0.0).

(a) Sketch the configuration on the right.



(b) Find the area of the region R.

$3/2$

(c) Evaluate the line integral $\int_{C_2} F(x,y) \bullet dr$, where $F(x,y) = \left(\cos\left(\frac{x^3+1}{x^2+1}\right), 2x + \frac{1}{y^2+1}\right)$.

$$\int_{y=1}^{y=0} \left(\quad \right) \cdot 0 + \left(0 + \frac{1}{y^2+1}\right) dy = \arctan y \Big|_{y=1}^{y=0}$$

$$= \arctan(0) - \arctan(1) = 0 - \pi/4 = -\pi/4$$

(d) Evaluate the line integral $\int_{C_1} F(x,y) \bullet dr$, where $F(x,y)$ is the function in part (b) **by using Green's Theorem only.** Other methods will not receive any credits.

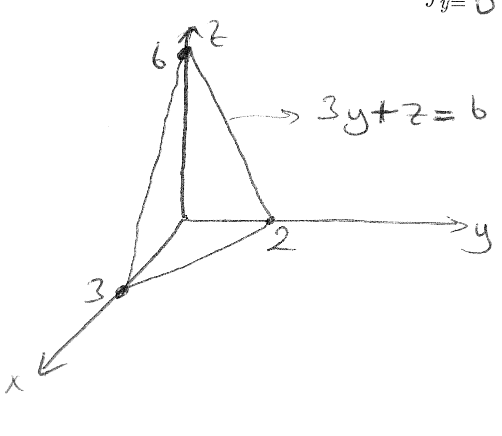
$$\int_{C_1 \cup C_2} F \cdot dr = \iint_R 2\, dA = 2 \cdot \iint_R dA = 2 \cdot Area(R)$$

$$= 2 \cdot \frac{3}{2} = 3$$

$$\int_{C_1} F \cdot dr = 3 - \int_{C_2} F \cdot dr = 3 - \left(\frac{-\pi}{4}\right) = 3 + \pi/4$$

3. (6 pts.) Consider the tetrahedron that is formed by the coordinate planes and the plane $2x+3y+z=6$. **Write bounds of the iterated triple integral** to calculate $\iiint_T x^2 z\, dV$ in the $dx\, dz\, dy$ order. **DO NOT EVALUATE THIS INTEGRAL.**

$$\int_{y=0}^{y=2} \int_{z=0}^{z=6-3y} \int_{x=0}^{x=\frac{6-3y-z}{2}} x^2 z\, dx\, dz\, dy$$



$3y+z=6 \iff z=6-3y$