M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 1	
Code : *Math 120* Acad.Year : *2015-2016* Semester : *Fall* Date : *04.11.2015* Time : *19:25* Duration : *25 minutes*	Last Name: Name: KEY Signature: 5 QUESTIONS 2 PAGES TOTAL 20 + 2 BONUS POINTS	List No: Student No:

pg1(10)	pg2(12)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. $(3 \times 2 = 6$ pts.) Suppose that $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 7$. Find

   (a) $\mathbf{a} \cdot (\mathbf{c} \times \mathbf{b}) = -7$

   as $\underset{\sim}{c} \times \underset{\sim}{b} = -(\underset{\sim}{b} \times \underset{\sim}{c})$

   (b) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0$

   as $\underset{\sim}{a} \times \underset{\sim}{b}$ is perpendicular to $\underset{\sim}{b}$

   (c) $\mathbf{a} \cdot ((2\mathbf{b} + \mathbf{c}) \times \mathbf{c}) = \underset{\sim}{a} \cdot (2\underset{\sim}{b} \times \underset{\sim}{c}) + \underset{\sim}{a} \cdot (\underset{\sim}{c} \times \underset{\sim}{c})$

   $= 2\underset{\sim}{a} \cdot (\underset{\sim}{b} \times \underset{\sim}{c}) + \underset{\sim}{a} \cdot (\underset{\sim}{c} \times \underset{\sim}{c})$

   $= 14 + 0 = 14$

2. $(4 \times 1 = 4$ pts.) Determine whether the given statements in Cartesian 3−space are true or false. Indicate your answers by circling **TRUE** or **FALSE**. No explanations required.

   (a) **TRUE** / **FALSE** Two lines perpendicular to a third line are parallel.

   (b) **TRUE** / **FALSE** Two planes perpendicular to a third plane are parallel.

   (c) **TRUE** / **FALSE** A plane and a line either intersect or are parallel.

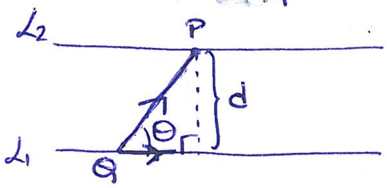
   (d) **TRUE** / **FALSE** Three points always determine a unique plane.

3. (4 pts.) Find the distance between the given lines.

$L_1 : x = -1 + 2t,\ y = -t,\ z = 3 + 3t$

$L_2 : x = 4 - 4s,\ y = -3 + 2s,\ z = -6s$

These two lines are parallel since $\underline{v}_2 = 2\underline{v}_1$,
where $\underline{v}_1 = (2,-1,3)$ and $\underline{v}_2 = (-4,2,-6)$ are the direction vectors
of L_1 and L_2, respectively.



$$d = \frac{|\vec{QP} \times \underline{v}_1|}{|\underline{v}_1|}.$$

Let $Q = (-1,0,3)$ and $P = (4,-3,0)$.
Then, $\vec{QP} = (4,-3,0) - (-1,0,3)$
$= (5,-3,-3)$.

$$\vec{QP} \times \underline{v}_1 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & -3 & -3 \\ 2 & -1 & 3 \end{vmatrix} = -12\underline{i} - 21\underline{j} + \underline{k}.$$

Hence, $$d = \frac{\sqrt{(-12)^2 + (-21)^2 + 1^2}}{\sqrt{2^2 + (-1)^2 + 3^2}} = \frac{\sqrt{586}}{\sqrt{14}}$$

4. (4 pts.) Find an equation of the plane that passes through the line of intersection of the planes $x+2y+z=5$ and $2x + y - z = 7$, and is perpendicular to the plane $3x - 2y + z = 4$.

The line of intersection has the direction vector $\underline{n}_1 \times \underline{n}_2$.

$$\underline{v} = \underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \underline{i}(-3) - \underline{j}(-3) + \underline{k}(-3) = -3\underline{i} + 3\underline{j} - 3\underline{k}$$

Since $\underline{n}_3 = (3,-2,1)$ is parallel to our plane, the normal vector of our plane is

$$\underline{v} \times \underline{n}_3 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3 & 3 & -3 \\ 3 & -2 & 1 \end{vmatrix} = \underline{i}(-3) + \underline{j}(6) + \underline{k}(-3) = -3\underline{i} - 6\underline{j} - 3\underline{k}.$$

find a point lying on the plane by letting $z = 0$ (a point lying on the line of intersection)

$\left.\begin{array}{l} x + 2y = 5 \\ 2x + y = 7 \end{array}\right\}$ $x = 3, y = 1$. $(3,1,0)$

So plane has the eqn:
$-3(x-3) - 6(y-1) - 3z = 0 \Rightarrow -3x - 6y - 3z = -15$

5. (4 pts.) Find parametric equations for the path of a particle in 3−space that moves around the unit circle in the xy−plane twice in a clockwise direction.

$x = \cos t$
$y = -\sin t$
$z = 0$, $0 \leq t \leq 4\pi$