M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables **Short Exam 3**	
Code : *Math 120* Acad.Year: *2011-2012* Semester : *Fall* Date : *06.12.2011* Time : *17:45* Duration : *45 minutes*	Last Name: Department: Section:	Name: Student No: Signature:
	3 QUESTIONS ON 2 PAGES TOTAL 32 POINTS	

1	2	3			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!

Please do not write on your desk!

1. (8 pts.) Calculate $\displaystyle\iint_R \frac{y}{x^2+y^2}\,dA$ where $R=[0,2]\times[1,4]$.

$$I=\int_0^2\int_1^4 \frac{y\,dy}{x^2+y^2}dx \quad \left| \begin{array}{l} u = x^2+y^2 \\ du = 2y\,dy \end{array} \right| \quad I=\int_0^2 \frac{du}{2\cdot u}\,dx = \int_0^2 \frac{\ln u}{2}\Big|_1^4 dx = \frac{1}{2}\int_0^2 \left(\ln(x^2+16)-\ln(x^2+1)\right)dx$$

$$\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - \int \frac{2x^2}{x^2+a^2}dx = x\ln(x^2+a^2) - \int 2\,dx + 2a^2\int\frac{dx}{x^2+a^2}$$

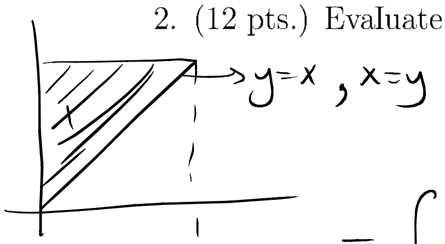
$$\boxed{\begin{array}{l|l} u=\ln(x^2+a^2) & dv = dx \\ du = \dfrac{2x}{x^2+a^2}dx & v = x \end{array}} \quad = x\ln(x^2+a^2) - 2x + 2\frac{a^2}{a^2}\int\frac{dx}{\left(\frac{x}{a}\right)^2+1} \qquad \begin{array}{l} p = x/a \\ dp = dx/a \end{array}$$

$$= x\ln(x^2+a^2) - 2x + 2a\int\frac{dp}{p^2+1} = x\ln(x^2+a^2) - 2x + 2a\arctan(x/a)$$

$$I = \frac{1}{2}\left[\begin{array}{l} \left(x\ln(x^2+16) - 2x + 2\cdot 4\cdot\arctan(x/4)\right)\Big|_0^2 \\ -\left(x\ln(x^2+1) - 2x + 2\cdot 1\cdot\arctan(x)\right)\Big|_0^2 \end{array}\right] = \frac{1}{2}\left[\begin{array}{l} 2\ln(20) + 8\arctan(1/2) \\ -\left(2\ln(5) + 2\arctan 2\right. \end{array}\right]$$

$$= \ln 20 - \ln 5 + 4\arctan\left(\tfrac{1}{2}\right) - 2\arctan 2$$

2. (12 pts.) Evaluate the integral by reversing the order of integration.



$$\int_0^1\int_x^1 e^{2x/3y}\,dydx = \int_0^1\int_0^y e^{2x/3y}\,dx\,dy$$

$$= \int_0^1 \frac{e^{2x/3y}}{2/3y}\Bigg|_{x=0}^{x=y} dy = \int_0^1 (e^{2/3}-1)\frac{3y}{2}\,dy$$

$$= 3(e^{2/3}-1)\frac{y^2}{4}\Big|_0^1 = \frac{3}{4}(e^{2/3}-1)$$

3. (12 pts.) Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy dx$$

$y = \sqrt{2x - x^2}$; $y \geq 0$

$y^2 = 2x - x^2$

$x^2 - 2x + y^2 = 0$

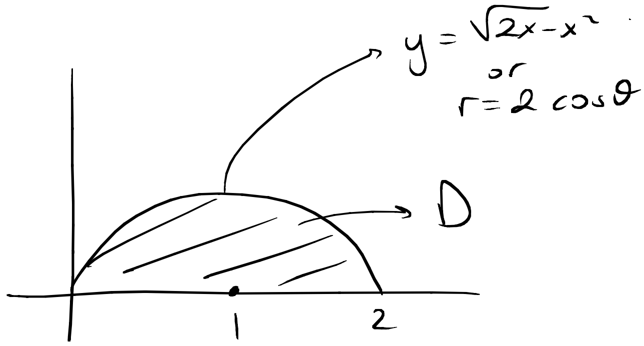
$x^2 - 2x + 1 + y^2 = 1$

$(x-1)^2 + y^2 = 1$

$x^2 + y^2 = 2x$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$



$$I = \int_0^{\pi/2} \int_0^{2\cos\theta} \sqrt{r^2} \cdot r\, dr\, d\theta = \int_0^{\pi/2} \int_0^{2\cos\theta} r^2\, dr\, d\theta$$

$$= \int_0^{\pi/2} \frac{r^3}{3} \Big|_0^{2\cos\theta} d\theta = \frac{8}{3} \int_0^{\pi/2} \cos^3\theta\, d\theta$$

$\left[u = \cos^2\theta, \ du = -2\cos\theta\sin\theta, \quad dv = \cos\theta\, d\theta, \ v = \sin\theta \right]$

$$I = \frac{8}{3}\left[\underbrace{\sin\theta\cos^2\theta \Big|_0^{\pi/2}}_{0} + 2\int_0^{\pi/2} \cos\theta \sin^2\theta\, d\theta \right] \qquad \left[\begin{array}{l} p = \sin\theta \\ dp = \cos\theta \end{array} \right]$$

$$= \frac{16}{3} \int_0^1 p^2\, d\theta = \frac{16}{3} \frac{p^3}{3} \Big|_0^1 = \frac{16}{3} \cdot \frac{1}{3} = \frac{16}{9}$$