M E T U
Northern Cyprus Campus

Calculus With Analytic Geometry Short Exam 1	
Code : *Math 119* Acad.Year: *2011-2012* Semester : *Spring* Date : *19.3.2011* Time : *17:45* Duration : *45 minutes*	Last Name: Name: Department: Student No: Section: Signature: Recitation:
	6 QUESTIONS ON 4 PAGES TOTAL 50 POINTS

1	2	3	4	5	6			
						SOLUTIONS		

Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1. (8 pts.) Evaluate the limit, if it exists.

(a) $\lim_{x\to -3} \dfrac{2x+6}{x^3+27} = \lim_{\substack{x\to -3\\ x\neq -3}} \dfrac{2\cancel{(x+3)}}{\cancel{(x+3)}(x^2-3x+9)} = \lim_{x\to -3} \dfrac{2}{x^2-3x+9} = \dfrac{2}{9+9+9}$

$= 2/27$

(b) $\lim_{x\to 1} \dfrac{\sin(x+1)}{x^2+2x+1} = \dfrac{\sin(2)}{1^2+2\cdot 1+1} = \dfrac{\sin(2)}{4}$

(c) $\lim_{x\to -\infty} \dfrac{x^3+3x}{\sqrt{4x^6+1}} = \lim_{x\to -\infty} \dfrac{x^3(1+3/x^2)}{|x^3|\sqrt{4+1/x^6}} = \lim_{x\to -\infty} \dfrac{x^3(1+3/x^2)}{-x^3\sqrt{4+1/x^6}}$

$= \lim_{x\to -\infty} \dfrac{-(1+\overbrace{3/x^2}^{\to 0})}{\sqrt{4+\underbrace{1/x^6}_{\to 0}}} = \dfrac{-1}{\sqrt{4}} = \dfrac{-1}{2}$

2. (8 pts.) Find the numbers at which $f(x)$ is discontinuous. **Give reasoning**.

$$f(x) = \begin{cases} x+3 & \text{if } x <= 1 \\ 4/x & \text{if } 1 < x < 4 \\ 2-x & \text{if } x >= 4 \end{cases}$$

$f(x)$ is continuous on $(-\infty, 1), (1,4), (4,\infty)$.

At $x=1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x+3 = 4$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 4/x = 4$

$f(1)=4$

$\Rightarrow$ cont. at $x=1$

At $x=4$

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 4/x = 1$

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} 2-x = -2$

$\lim_{x\to 4} f(x)$ d.n.e

Hence $f(x)$ is discontinuous only at $x=4$.

3. (8 pts.) Find $f'(3)$ if $f(x) = 5x^2 - 4$, *using the limit definition of the derivative only.* (Note : Any other methods will not receive *any* credit.)

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{(5(3^2+6h+h^2)-4)-(5\cdot 3^2-4)}{h}$$

$$= \lim_{h\to 0} \frac{5\cdot 3^2 + 30h + 5h^2 - 4 - 5\cdot 3^2 + 4}{h} = \lim_{h\to 0} \frac{h(30+5h)}{h}$$

$$= \lim_{h\to 0} 30 + 5h = 30$$

$$f'(3) = 30$$

4. (8 pts.) Find the tangent line to the graph of $y = 2\cos(x) + 3\sin(x)$ at $x = \frac{\pi}{3}$.

$$m_T = f'(\pi/3), \quad pt = \left(\frac{\pi}{3}, f(\pi/3)\right) = \left(\frac{\pi}{3}, 1 + \frac{3\sqrt{3}}{2}\right)$$

$$f'(x) = -2\sin x + 3\cos x \Rightarrow f'\left(\frac{\pi}{3}\right) = -2\cdot\frac{\sqrt{3}}{2} + 3\cdot\frac{1}{2} = \frac{3}{2} - \sqrt{3}$$

Equation of the tangent line:

$$y - \left(1 + \frac{3\sqrt{3}}{2}\right) = \left(\frac{3}{2} - \sqrt{3}\right)\cdot\left(x - \frac{\pi}{3}\right)$$

or

$$y = \left(\frac{3}{2} - \sqrt{3}\right)\left(x - \frac{\pi}{3}\right) + \left(1 + \frac{3\sqrt{3}}{2}\right)$$

5. (10 pts.) Find the following derivatives. **DO NOT SIMPLIFY YOUR ANSWERS.**

(a) $\frac{d}{du}\left(u^{\pi} - \frac{1}{\sqrt[5]{u^3}}\right) = \frac{d}{du}\left(u^{\pi} - u^{-3/5}\right)$

$= \pi \cdot u^{\pi - 1} + \frac{3}{5} u^{-8/5}$

(b) $\frac{d}{dt}\left(\frac{2t^2+6}{t^3+3}\right) = \frac{4t \cdot (t^3+3) - (2t^2+6)(3t^2)}{(t^3+3)^2}$

(c) $\frac{d}{dp}\left((19p+3)^{2012}\right) = \frac{dy}{dz}\frac{dz}{dp} = 2012 \cdot z^{2011} \cdot 19$

$19p+3 = z$

$z^{2012} = y$

$= 2012 \cdot 19 \cdot (19p+13)^{2011}$

(d) $\frac{d}{dx}\left(\sin(2x)\cos(x^2+1)\right) = \frac{d}{dx}(\sin(2x)) \cdot \cos(x^2+1) + \sin(2x) \cdot \frac{d}{dx}(\cos(x^2+1))$

$= 2\cos(2x) \cdot \cos(x^2+1) - \sin(2x) \cdot \sin(x^2+1) \cdot 2x$

6. (8 pts.) Using the definition of the limit, prove that $\lim_{x \to 3} x^2 - 1 = 8$.

Given $\varepsilon > 0$, want to find $\delta > 0$ so that

$$0 < |x-3| < \delta \implies |(x^2-1)-8| < \varepsilon$$

$$|x^2-1-8| = |x^2-9| = |x-3| \cdot |x+3| \underset{\text{want}}{<} \varepsilon$$

Let $|x-3| < 1 \iff 2 < x < 4 \implies 5 < x+3 < 7$

$$\implies |x+3| < 7$$

Pick $\delta < \min(1, \varepsilon/7)$

Then if $0 < |x-3| < \delta$

I $|x-3| < 1$, so $|x+3| < 7$

and

II $0 < |x-3| < \delta$, so $|x-3| < \varepsilon/7$

Hence,

$$|(x^2-1)-8| = |x-3| \cdot |x+3| \underset{\mathrm{I}}{<} |x-3| \cdot 7 \underset{\mathrm{II}}{<} \frac{\varepsilon}{7} \cdot 7 = \varepsilon$$

q.e.d.