METU
Northern Cyprus Campus

Applied Mathematics for Engineers
Midterm

Code : *Math 210*
Acad.Year : *2011-2012*
Semester : *Spring*
Date : *19.4.2012*
Time : *17:40*
Duration : *120 minutes*

Last Name:
Name :
Department: KEY
Signature:

Student No:
Section:

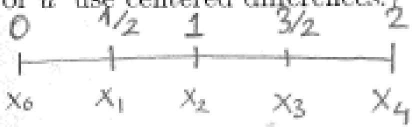
7 QUESTIONS ON 6 PAGES
TOTAL 100 POINTS

1 (12)	2 (22)	3 (10)	4 (16)	5 (12)	6 (14)	7 (14)	

Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1.(12 pts) Consider the differential equation $xu'' + u' = \delta(x-\frac{1}{2}) + 2\delta(x-\frac{3}{2})$ $\quad u(0) = u(2) = 0$. Write the matrix equation whose solution would give an approximate answer the the differential equation with $h = \frac{1}{2}$. You will not get any extra points for solving the system – just write all of the matrices. (For u' use centered differences.)

$\Delta x = h = \frac{1}{2}$



$i = 0, 1, 2, 3, 4$

$U(0) = U(x_0) = U_0 = 0$

$U(2) = U(x_4) = U_4 = 0$

$$u'' \approx \frac{U(x_{i-1}) - 2U(x_i) + U(x_{i+1})}{h^2} = \frac{U_{i-1} - 2U_i + U_{i+1}}{h^2}$$

$$u' \approx \frac{U(x_{i+1}) - U(x_{i-1})}{2\cdot h} = \frac{U_{i+1} - U_{i-1}}{2h}$$

~~$i=0$~~ Not possible since there's no U_{-1}.

$i=1$ $\quad x_1 \cdot \frac{(\overset{0}{\overset{\shortparallel}{U_0}} - 2U_1 + U_2)}{h^2} + \frac{U_2 - \overset{0}{\overset{\shortparallel}{U_0}}}{2h} = \frac{1}{h}\cdot 1$ ← impulse at $\frac{1}{2} = x_1$

$i=2$ $\quad x_2 \cdot \frac{(U_1 - 2U_2 + U_3)}{h^2} + \frac{U_3 - U_1}{2h} = 0$

$i=3$ $\quad x_3 \cdot \frac{(U_2 - 2U_3 + \overset{0}{\overset{\shortparallel}{U_4}})}{h^2} + \frac{\overset{0}{\overset{\shortparallel}{U_4}} - U_2}{2h} = 2\cdot\frac{1}{h}\cdot 1$ ← impulse at $\frac{3}{2} = x_3$

~~$i=4$~~ Not possible since there's no U_5.

Hence, we get:

$$\frac{1}{(\frac{1}{2})^2}\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{3}{2}\end{bmatrix}\begin{bmatrix}-2 & 1 & 0\\ 1 & -2 & 1\\ 0 & 1 & -2\end{bmatrix}\begin{bmatrix}U_1\\ U_2\\ U_3\end{bmatrix} + \frac{1}{2\cdot\frac{1}{2}}\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}U_1\\ U_2\\ U_3\end{bmatrix} = \frac{1}{\frac{1}{2}}\begin{bmatrix}1\\ 0\\ 2\end{bmatrix}$$

2.(9+5+3+3+2 pts) Let A be the symmetric matrix $\begin{bmatrix}2&1&0\\1&2&0\\0&0&1\end{bmatrix}$

(a) Find the eigenvalues and corresponding eigenvectors of A.

$$\det(A-\lambda I) = \begin{vmatrix}2-\lambda&1&0\\1&2-\lambda&0\\0&0&1-\lambda\end{vmatrix} = (1-\lambda)\big((2-\lambda)(2-\lambda)-1\big) = (1-\lambda)(\lambda-1)(\lambda-3)$$

$\lambda=1$

$$\begin{bmatrix}1&1&0\\1&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}1&1&0\\1&1&0\\0&0&0\end{bmatrix}\xrightarrow{-R_1+R_2}\begin{bmatrix}\overset{\text{pivot}}{\boxed{1}}&1&0\\0&0&0\\0&0&0\end{bmatrix}$$

$v_1+v_2=0$

$v_1=-v_2$

$v_2=v_2$

$v_3=v_3$

so $\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=v_2\begin{bmatrix}-1\\1\\0\end{bmatrix}+v_3\begin{bmatrix}0\\0\\1\end{bmatrix}$

$\lambda=3$

$$\begin{bmatrix}-1&1&0\\1&-1&0\\0&0&-2\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}-1&1&0\\1&-1&0\\0&0&-2\end{bmatrix}\xrightarrow{R_1+R_2}\begin{bmatrix}\overset{\text{pivot}}{\boxed{-1}}&1&0\\0&0&0\\0&0&\underset{\text{pivot}}{\boxed{-2}}\end{bmatrix}$$

$\left.\begin{matrix}-v_1+v_2=0\\-2v_3=0\end{matrix}\right\}\begin{matrix}v_1=v_2\\v_3=0\\v_2=v_2\end{matrix}$

so $\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}=v_2\begin{bmatrix}1\\1\\0\end{bmatrix}$

(b) Give $Q\Lambda Q^T$ decomposition of A where Q is an orthogonal matrix and Λ is a diagonal matrix.

We need to make eigenvectors unit length (norm). Then we get

$$\begin{bmatrix}2&1&0\\1&2&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{-1}{\sqrt2}&0&\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}&0&\frac{1}{\sqrt2}\\0&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&3\end{bmatrix}\begin{bmatrix}\frac{-1}{\sqrt2}&\frac{1}{\sqrt2}&0\\0&0&1\\\frac{1}{\sqrt2}&\frac{1}{\sqrt2}&0\end{bmatrix}$$

$$A \quad = \quad Q \quad\cdot\quad \Lambda \quad\cdot\quad Q^T$$

Note: $Q^T=Q^{-1}$

since Q is an

orthogonal matrix.

(c) Compute A^{101}. (You can leave your answer as two product of matrices, not as powers.)

$$A^{101}=(Q\cdot\Lambda\cdot Q^T)^{101}=\underbrace{Q\cdot\Lambda\cdot\overbrace{Q^T\cdot Q}^{I}\cdot\Lambda\cdot\overbrace{Q^T\cdots}^{I}\cdots\overbrace{Q^T Q}^{I}\Lambda Q^T}_{101-\text{times}}$$

$$Q^T=Q^{-1}$$

$$=Q\,\Lambda^{101}\,Q^T=\begin{bmatrix}\frac{-1}{\sqrt2}&0&\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}&0&\frac{1}{\sqrt2}\\0&1&0\end{bmatrix}\cdot\begin{bmatrix}1^{101}&0&0\\0&1^{101}&0\\0&0&3^{101}\end{bmatrix}\begin{bmatrix}\frac{-1}{\sqrt2}&\frac{1}{\sqrt2}&0\\0&0&1\\\frac{1}{\sqrt2}&\frac{1}{\sqrt2}&0\end{bmatrix}$$

(d) Write energy function $E(x) = x^TAx$ of A as sum of three squares by using $Q^T\Lambda Q$ decomposition of A.

$$[x_1\ x_2\ x_3]\begin{bmatrix}2&1&0\\1&2&0\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \underbrace{[x_1\ x_2\ x_3]\begin{bmatrix}-\frac{1}{\sqrt{2}}&0&\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}&0&\frac{1}{\sqrt{2}}\\0&1&0\end{bmatrix}}_{(Q^Tx)^T}\begin{bmatrix}1&0&0\\0&1&0\\0&0&3\end{bmatrix}\underbrace{\begin{bmatrix}-\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}&0\\0&0&1\\ \frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}}_{(Q^Tx)}$$

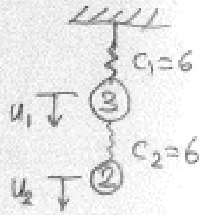
$$= \begin{bmatrix}\frac{1}{\sqrt{2}}(x_2-x_1) & x_3 & \frac{1}{\sqrt{2}}(x_1+x_2)\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&3\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}(x_2-x_1)\\x_3\\ \frac{1}{\sqrt{2}}(x_1+x_2)\end{bmatrix} = \frac{1}{2}(x_2-x_1)^2 + 1.x_3^2 + 3\cdot\frac{1}{2}(x_1+x_2)^2$$

(e) Is A positive-definite ? Explain.

Yet, it's positive-definite, since all of its eigenvalues 1,1,3 are positive.

3.(5+5 pts) Consider the spring-mass system with two masses $m_1 = 3, m_2 = 2$ and two springs with spring constants $c_1 = c_2 = 6$, where top end is fixed and bottom end is free.

(a) Compute the stiffness matrix K. Show your work.



$e_1 = u_1$

$e_2 = u_2 - u_1$

$$\begin{bmatrix}e_1\\e_2\end{bmatrix} = \underbrace{\begin{bmatrix}1&0\\-1&1\end{bmatrix}}_{A}\begin{bmatrix}u_1\\u_2\end{bmatrix} \qquad K = A^T\cdot C\cdot A$$

$$K = \begin{bmatrix}1&-1\\0&1\end{bmatrix}\begin{bmatrix}6&0\\0&6\end{bmatrix}\begin{bmatrix}1&0\\-1&1\end{bmatrix} = \begin{bmatrix}12&-6\\-6&6\end{bmatrix}$$

(b) If the system is in equilibrium, what will be the elongation of each spring?

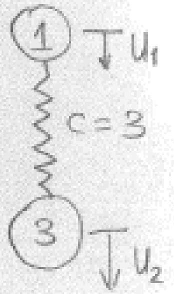
In equilibrium $K\cdot u = f$ where $f = \begin{bmatrix}m_1g\\m_2g\end{bmatrix} = \begin{bmatrix}3g\\2g\end{bmatrix} = g\begin{bmatrix}3\\2\end{bmatrix}$

$$u = K^{-1}\cdot f = \frac{1}{36}\begin{bmatrix}6&+6\\6&12\end{bmatrix}g\cdot\begin{bmatrix}3\\2\end{bmatrix} = g\cdot\begin{bmatrix}\frac{5}{6}\\ \frac{7}{6}\end{bmatrix}$$

4.(14+2 pts) Consider the free/free spring system consisting of two masses $m_1 = 1$ and $m_2 = 3$ connected by a spring with spring constant $c = 3$ with no external forces present.

(a) Find $u_1(t)$ and $u_2(t)$ (the position of the two masses as functions of time), if the masses start at position $u_1(0) = 6$, $u_2(0) = -2$ and velocity $u_1'(0) = 6$, $u_2'(0) = -2$.

$e = u_2 - u_1$



$$e = \underbrace{[-1 \ 1]}_{A}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad K = A^T \cdot C \cdot A = \begin{bmatrix} -1 \\ 1 \end{bmatrix}[3][-1 \ \ 1] = 3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$M^{-1} \cdot K = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}$$

$$\det(M^{-1} \cdot K - \lambda I) = \begin{vmatrix} 3-\lambda & -3 \\ -1 & 1-\lambda \end{vmatrix} = \lambda^2 - 4\lambda = \lambda(\lambda - 4) = 0 \quad \begin{matrix} \lambda = 0 \\ \lambda = 4 \end{matrix}$$

$\lambda = 0$

$$\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \xrightarrow[\frac{1}{3}R_1]{\frac{1}{3}R_1 + R_2} \begin{bmatrix} \overset{\text{pivot}}{\boxed{1}} & -1 \\ 0 & 0 \end{bmatrix} \qquad \begin{matrix} v_1 - v_2 = 0 \\ v_2 = v_2 \end{matrix} \qquad \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda = 4$

$$\begin{bmatrix} -1 & -3 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} -1 & -3 \\ -1 & -3 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} -1 & -3 \\ 0 & 0 \end{bmatrix} \qquad \begin{matrix} -v_1 - 3v_2 = 0 \\ v_2 = v_2 \end{matrix} \qquad \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = v_2\begin{bmatrix} -3 \\ +1 \end{bmatrix}$$

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = (a_1 + b_1 \cdot t)\begin{bmatrix} 1 \\ 1 \end{bmatrix} + (a_2\cos(2t) + b_2\sin(2t))\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} u_1(0) \\ u_2(0) \end{bmatrix} = a_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + a_2\begin{bmatrix} -3 \\ 1 \end{bmatrix} \Rightarrow a_1 = 0 \quad a_2 = -2.$$

$$\begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} u_1'(0) \\ u_2'(0) \end{bmatrix} = b_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2b_2\begin{bmatrix} -3 \\ 1 \end{bmatrix} \Rightarrow b_1 = 0, \ b_2 = -1$$

(b) Explain the movement of the masses.

$$u(t) = (-2\cos(2t) - \sin(2t))\begin{bmatrix} -3 \\ 1 \end{bmatrix} = (2\cos(2t) + \sin(2t))\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Both of the masses are bouncing in opposite directions. Top mass bounces (moves) more.

5. (3+3+3+3 pts) This problem has four unrelated parts about eigenvalues, eigenvectors and positive-definite matrices.

(a) Let A have eigenvalues -2 and 3 with corresponding eigenvectors $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, respectively. Compute $A \cdot v$ where $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = -1 \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 1 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad A \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = A\left(-1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = -1 \cdot (-2)\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

(b) Given $\lim_{k \to \infty} A^k = O$ for a symmetric 2×2 matrix A. Which of the following matrices can be A? Explain. (**Hint:** Consider $A = S \cdot D \cdot S^{-1}$)

$$\begin{bmatrix} 2 & \frac{3}{2} \\ \frac{2}{3} & 2 \end{bmatrix} \begin{matrix} \lambda_1 = 1 \\ \lambda_2 = 3 \end{matrix} \qquad \boxed{\begin{bmatrix} \frac{1}{2} & 0 \\ 1 & \frac{2}{3} \end{bmatrix} \begin{matrix} \lambda_1 = \frac{1}{2} \\ \lambda_2 = \frac{2}{3} \end{matrix}} \checkmark \qquad \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{matrix} \lambda_1 = -1 \\ \lambda_2 = 2 \end{matrix}$$

$A^k = S \cdot D^k \cdot S^{-1}$. If $\lim_{k \to \infty} A^k = 0$, then $\lim_{k \to \infty} D^k$ must be 0 zero matrix

since D is matrix of eigenvalues, D^k is the k^{th} power of each eigenvalue along the diagonal. If $D^k \to 0$ as $k \to \infty$, eigenvalues must have absolute value less than 1. Hence, answer is the second one.

(c) Given $K = A^T A$ where $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & c \end{bmatrix}$. Find the value(s) of c so that K is positive-definite.

K is positive-definite if and only if $\text{Null}(A) = \{0\}$ We will solve $Au = 0$

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & c \end{bmatrix} \xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 1 & c \end{bmatrix} \xrightarrow{-R_2 + R_3} \begin{bmatrix} \textcircled{1} & 0 & 2 \\ 0 & \textcircled{1} & 3 \\ 0 & 0 & \textcircled{c-3} \end{bmatrix}$$

If we have 3 pivots $\text{Null}(A) = \{0\}$

Hence, If $c \neq 3$, then K is positive-definite.

(d) Show that any positive-definite matrix A has positive entries along its diagonal.
(**Hint:** Compute the energy of standard basis vectors.)

$$\begin{bmatrix} 0 \cdots 0 & 1 & 0 \cdots 0 \end{bmatrix} \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow i^{th} = a_{ii} = \text{Energy}\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) > 0 \quad \text{So, } a_{ii} > 0$$

$\uparrow$
i^{th}

6.(4+3+3+2+2 pts)The following parts involve the matrix $A = \begin{bmatrix}2&4\\4&3\end{bmatrix}$.

(a) Find the LU decomposition of A.

$$\begin{bmatrix}2&4\\4&3\end{bmatrix}\xrightarrow{-2R_1+R_2}\begin{bmatrix}2&4\\0&-5\end{bmatrix}=U$$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\xrightarrow{2R_1+R_2}\begin{bmatrix}1&0\\2&1\end{bmatrix}=E_1=L$$

$$\begin{bmatrix}2&4\\4&3\end{bmatrix}=\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}2&4\\0&-5\end{bmatrix}$$

$$A \;=\; L\cdot U$$

(b) Use the LU decomposition from **(a)** to solve $Ax = \begin{bmatrix}1\\-3\end{bmatrix}$

$$Ax=LU\cdot x=\begin{bmatrix}+1\\-3\end{bmatrix} \qquad U\cdot x=y$$

$$L\cdot y=\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}1\\-3\end{bmatrix}\Rightarrow\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}1\\-5\end{bmatrix}$$

$$U\cdot x=\begin{bmatrix}2&4\\0&-5\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\-5\end{bmatrix}\Rightarrow\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-\frac{3}{2}\\1\end{bmatrix}$$

(c) Find the LDL^T decomposition of A.

pivots

$$A=\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}2&4\\0&-5\end{bmatrix}=\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}2&0\\0&-5\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}$$

$$L\cdot U \qquad\qquad L\cdot D\cdot L^T$$

(d) Compute the determinant of A by using **(c)**.

determinant is equal to the product pivots.

$$\det(A)=2\cdot(-5)=-10.$$

(d) Is A positive definite? Explain.

No, it's not. Since one pivot (2) is positive, but the other pivot (−5) is negative. It's indefinite.

7.(10+4 pts) Use the method of least squares to find the best fitting formula of the form $f(x) = ax^2 + b$ to the data below, and compute the error, $e^T e$.

x	f
0	2
1	4
-1	-3

$$2 = f(0) = a \cdot 0^2 + b$$
$$4 = f(1) = a \cdot 1^2 + b \Rightarrow \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{u} = \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix} = b$$
$$-3 = f(-1) = a \cdot (-1)^2 + b$$

Then, $K = A^T \cdot A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$

$$f = A^T \cdot b = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\hat{u} = K^{-1} \cdot f = \frac{1}{2} \begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} \\ 2 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\boxed{f(x) = -\frac{3}{2} x^2 + 2}$$

$$e = b - A\hat{u} = \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{3}{2} \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix} - \begin{bmatrix} 2 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{7}{2} \\ -\frac{7}{2} \end{bmatrix}$$

$$\text{Error} = e^T \cdot e = \boxed{\frac{98}{4}}$$