M E T U
Northern Cyprus Campus

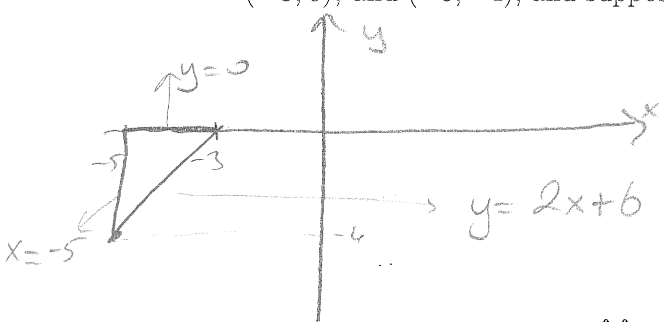
	Calculus for Functions of Several Variables Short Exam 2	

Code : *Math 120* Acad.Year : *2012-2013* Semester : *Spring* Date : *15.04.2013* Time : *17:45* Duration : *35 minutes*	Last Name: Name: Student No: Signature: 4+1 QUESTIONS ON 2 PAGES TOTAL 42+4=46 POINTS

1	2	3	4	5	KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. ($2 \times 6 = 12$ pts.) Let T be the triangle in the 2−dimensional space with vertices $(-5, 0)$, $(-3, 0)$, and $(-5, -4)$, and suppose $f(x, y)$ is a continuous function on T.



$y = 2x + 6 \longleftrightarrow x = \dfrac{y-6}{2}$

(a) Find $\alpha, \beta, \gamma, \theta$ so that $\displaystyle\iint_T f(x, y)\, dA = \int_\alpha^\beta \int_\gamma^\theta f(x, y)\, dy\, dx$

$\alpha = -5$; $\beta = -3$; $\gamma = 2x+6$; $\theta = 0$

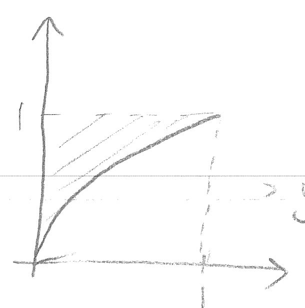
DO NOT EVALUATE THIS INTEGRAL.

(b) Find $\alpha, \beta, \gamma, \theta$ so that $\displaystyle\iint_T f(x, y)\, dA = \int_\alpha^\beta \int_\gamma^\theta f(x, y)\, dx\, dy$

$\alpha = -4$; $\beta = 0$; $\gamma = -5$; $\theta = \dfrac{y-6}{2}$

DO NOT EVALUATE THIS INTEGRAL.

2. (8 pts.) Evaluate the integral $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3)\, dy\, dx$ by reversing the order of integration.



$y = \sqrt{x} \longleftrightarrow x = y^2$

$$\int_{y=0}^{y=1} \int_{x=0}^{x=y^2} \cos(y^3)\, dx\, dy$$

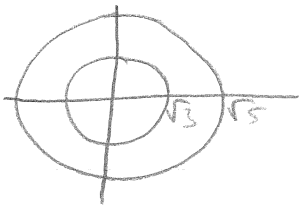
$$= \int_{y=0}^{y=1} y^2 \cos y^3\, dy$$

$u = y^3$
$du = 3y^2 dy$

$$= \int_0^1 \cos u \frac{du}{3} = \frac{1}{3}(\sin u)\Big|_0^1 = \frac{1}{3}(\sin 1 - \sin 0) = \frac{\sin(1)}{3}$$

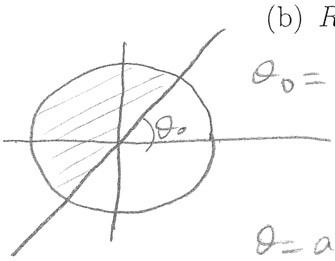
3. ($3 \times 4 = 12$ pts.) Convert the integral $\iint_R f(x, y)\, dA$ to an integral in **polar coordinates**.

(a) R is the annulus (washer) between the circles $x^2 + y^2 = 3$ and $x^2 + y^2 = 5$.



$$\int_{\theta=0}^{\theta=2\pi} \int_{r=\sqrt{3}}^{r=\sqrt{5}} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

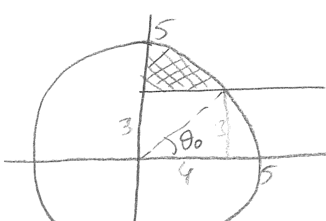
(b) R is the region inside the circle $x^2 + y^2 = 4$ that is above the line $y = 2x$.



$\theta_0 = \arctan(2)$

$$\int_{\theta=\arctan 2}^{\theta=\arctan 2 + \pi} \int_{r=0}^{r=2} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

(c) R is the region in the first quadrant inside the circle $x^2 + y^2 = 25$ that is ~~to the~~ above the line $y = 3$.



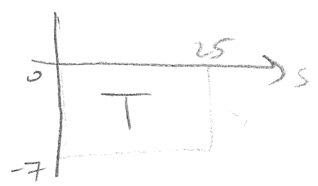
$y = 3 \Leftrightarrow r\sin\theta = 3 \Leftrightarrow r = \frac{3}{\sin\theta}$

$\theta_0 = \arctan\left(\frac{3}{4}\right)$

$$\int_{\theta=\arctan(3/4)}^{\theta=\pi/2} \int_{r=\frac{3}{\sin\theta}}^{r=5} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

4. (10 pts.) Use the change of variables $s = y$, $t = y - x^2$ to evaluate $\iint_R x\, dx\, dy$ over the region R in the first quadrant bounded by $y = 0$, $y = 25$, $y = x^2$, and $y = x^2 - 7$.

$s = 0$ $\quad s = 25$ $\quad y - x^2 = t = 0$ $\quad y - x^2 = t = -7$



$$\frac{\partial(x,y)}{\partial(s,t)} = \frac{1}{\frac{\partial(s,t)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} 0 & 1 \\ -2x & 1 \end{vmatrix}} = \frac{1}{2x}$$

$$\int_{s=0}^{s=25} \int_{t=-7}^{t=0} x \cdot \left|\frac{1}{2x}\right| dt\, ds$$

$x > 0$ since we are in the first quad.

$$= \frac{1}{2} \int_{s=0}^{s=25} \int_{t=-7}^{t=0} dt\, ds$$

$$= \frac{1}{2} \text{Area}(T) = \frac{1}{2} \cdot 25 \cdot 7$$

5. (Bonus) (4 pts.) A curve with polar equation $r = \dfrac{120}{\cos(\theta) + 20\sin(\theta)}$ represents a line. Find m and b so that this line is $y = mx + b$ in Cartesian Coordinates.

$r\cos\theta + 20 r\sin\theta = 120$

$x + 20y = 120$

$y = \frac{120 - x}{20} = \frac{-1}{20} x + 6$

$m = \frac{-1}{20}$

$b = 6$