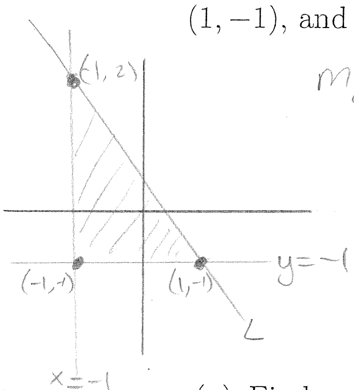
M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 2	
Code : *Math 120* Acad.Year: *2012-2013* Semester : *Fall* Date : *12.12.2012* Time : *18:45* Duration : *45 minutes*	Last Name: Department: Section: Recitation: 5 QUESTIONS ON 2 PAGES TOTAL 42+2 POINTS	Name: Student No: Signature:

1	2	3	4	5			
					KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. ($2\times 6 = 12$ pts.) Let D be the triangle in the 2−dimensional space with vertices $(-1,-1)$, $(1,-1)$, and $(-1,2)$, and suppose $f(x,y)$ be a function continuous function on D.



$m_L = \frac{-3}{2}$ $L: \frac{y-(-1)}{x-1} = -\frac{3}{2}$

$3x + 2y = 1$

$y = \frac{1-3x}{2}$ $x = \frac{1-2y}{3}$

(a) Find $\alpha, \beta, \gamma, \theta$ so that $\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_\gamma^\theta f(x,y)\,dy\,dx$

$\alpha = -1$; $\beta = 1$; $\gamma = -1$; $\theta = \frac{1-3x}{2}$

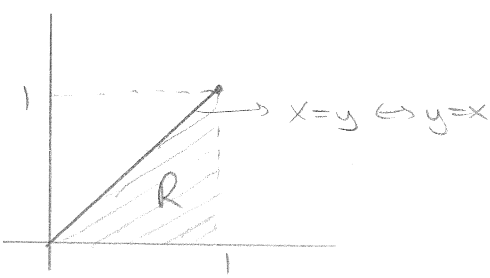
DO NOT EVALUATE THIS INTEGRAL.

(b) Find $\alpha, \beta, \gamma, \theta$ so that $\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_\gamma^\theta f(x,y)\,dx\,dy$

$\alpha = -1$; $\beta = 2$; $\gamma = -1$; $\theta = \frac{1-2y}{3}$

DO NOT EVALUATE THIS INTEGRAL.

2. (8 pts.) Evaluate the integral $\int_0^1 \int_y^1 \sin(x^2)\,dx\,dy$ by reversing the order of integration.



$I = \iint_R \sin(x^2)\,dA$

$= \int_{x=0}^{x=1} \int_{y=0}^{y=x} \sin(x^2)\,dy\,dx$

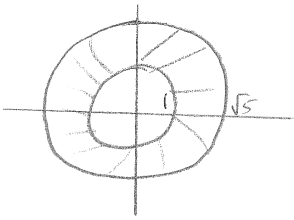
$= \int_{x=0}^{x=1} x\cdot\sin(x^2)\,dx = \frac{1}{2}\int_0^1 \sin u\,du = -\frac{1}{2}\cos u\Big|_0^1$

$u = x^2$
$du = 2x\,dx$

$= -\frac{1}{2}(\cos(1) - 1) = \boxed{\frac{1-\cos(1)}{2}}$

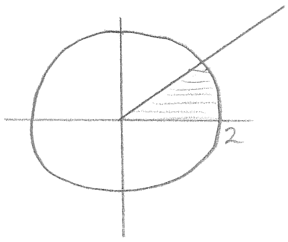
3. (3×4 = 12 pts.) Convert the integral $\iint_R f(x,y)\,dA$ to an integral in **polar coordinates**.

(a) R is the annulus (washer) between the circles $x^2+y^2=1$ and $x^2+y^2=5$.



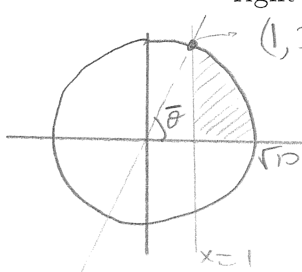
$$\int_{\theta=0}^{\theta=2\pi}\int_{r=1}^{r=\sqrt{5}} f(r\cos\theta, r\sin\theta)\cdot r\,dr\,d\theta$$

(b) R is the region in the first quadrant inside the circles $x^2+y^2=4$ that is below the line $y=x$.



$$\int_{\theta=0}^{\theta=\pi/4}\int_{r=0}^{r=2} f(r\cos\theta, r\sin\theta)\; r\,dr\,d\theta$$

(c) R is the region in the first quadrant inside the circles $x^2+y^2=10$ that is to the right of the line $x=1$.

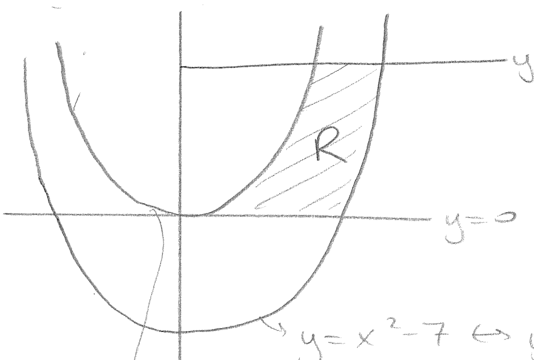


$(1,3)$ (This point is on the circle and $x=1$ & $y\geq 0$)

$\tan\bar\theta = \frac{3}{1} \Rightarrow \bar\theta = \arctan(3)$ Also $x=1 \Leftrightarrow r\cos\theta = 1 \Leftrightarrow r = 1/\cos\theta$

$$\int_{\theta=0}^{\theta=\arctan 3}\int_{r=1/\cos\theta}^{r=\sqrt{10}} f(r\cos\theta, r\sin\theta)\; r\,dr\,d\theta$$

4. (10 pts.) Use the change of variables $s=y$, $t=y-x^2$ to evaluate $\iint_R x\,dx\,dy$ over the region R in the first quadrant bounded by $y=0$, $y=25$, $y=x^2$, and $y=x^2-7$.



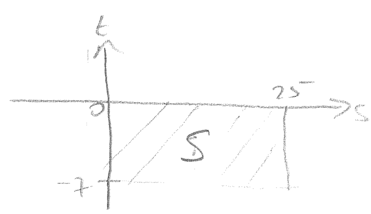
$y=x^2-7 \Leftrightarrow y-x^2=-7$

$y=x^2 \Leftrightarrow y-x^2=0$

$$\frac{\partial(x,y)}{\partial(s,t)} = \frac{1}{\frac{\partial(s,t)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} 0 & 1 \\ -2x & 1 \end{vmatrix}} = \frac{1}{2x}$$

$0 \leq s \leq 25$

$-7 \leq t \leq 0$



$$\iint_R x\,dx\,dy = \iint_S x\cdot\frac{1}{2x}\,ds\,dt = \frac{1}{2}\iint_S ds\,dt = \frac{1}{2}\text{ Area of } S$$

$$= \frac{1}{2}\cdot 7\cdot 25$$

5. (2 pts.) What is the full e-mail address of your teaching assistant?

abozer@metu.edu.tr