M E T U

Northern Cyprus Campus

Math 210	**Applied mathematics for engineers**	**I. Exam**	**30.03.2009**
Last Name : Name : Student No:	Dept./Sec. : Time : 17: 40 Duration : 120 *minutes*	Signature	
4 QUESTIONS ON 4 PAGES		TOTAL 100 POINTS	

1	2	3	4			

Q1 (12+13=25 pts.) (a) Consider the system

$$\begin{aligned} x + 2y &= 2b \\ x + 4y &= 2 \\ 2x + 5y &= 0. \end{aligned}$$

Find the value of b such that this system has a unique solution. Then, find the solution.

(b) Consider the system

$$\begin{aligned} x + y + 2z &= 1 \\ 2x + 3y &= 1 \\ bx + 3y + 3z &= a. \end{aligned}$$

Find the values of a and b so that the system has infinitely many solutions. Then, find the solution set.

Q2 (25 pts.) Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) Find the determinant of A by triangulating the matrix.

(b) Find the inverse of A using the Gauss-Jordan method.

Q3 (25 pts.) Answer the following (independent) questions. Assume that A and B are $n \times n$ matrices, and $\mathbf{x}$ is an $n \times 1$ vector. Give reasons (proofs or counterexamples) supporting your answers.

(a) Suppose that $A^2 = B^2$. Is it necessarily true that $A = B$ or $A = -B$?

(b) Consider the set of vectors $V = \{(v_1, v_2) \in \mathbb{R}^2 | v_1 - v_2 = 10\}$. Is V a vector space?

(c) If $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions, does $BA\mathbf{x} = \mathbf{0}$ always have infinitely many solutions?

(d) If $A\mathbf{x} = \mathbf{0}$ has a unique solution, does $BA\mathbf{x} = \mathbf{0}$ always have a unique solution?

(e) Is it true that $rank(AB) = rank(B^T A^T)$?

Q4 (25 pts.) Let

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 11 \\ 3 & 0 & 6 \\ 4 & 2 & 14 \\ -1 & 5 & 13 \end{bmatrix}$$

Find the rank of A, a basis for the row space of A, and a basis for the column space of A.