M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 1	
Code : *Math 120* Acad.Year: *2011-2012* Semester : *Summer* Date : *19.7.2012* Time : *18:40* Duration : *35 minutes*	Last Name: Name: Department: Student No: Section: Signature: Recitation:	
	6 QUESTIONS ON 2 PAGES TOTAL 45 POINTS	

1	2	3	4	5	6	ABOZER		

Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1. (1 pt.) What is the office number of your professor?

2. ($4 \times 4 = 16$ pts.) Find the limit, if it exists and prove your claim. Otherwise, show that the limit does not exist.

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{e^{x^2+y^2}}{19x^2+07y^2}$

   $\lim_{(x,y)\to(0,0)} e^{x^2} = 1$
   $\lim_{(x,y)\to(0,0)} 19x^2+07y^2 = 0$
   } limit does not exist.

   (b) $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^4+y^4}$

   $\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \dfrac{x^2y^2}{x^4+y^4} = \lim_{(x,y)\to(0,0)} \dfrac{0}{y^4} = 0$ | $\lim_{\substack{(x,y)\to(0,0)\\ y=x}} \dfrac{x^2y^2}{x^4+y^4} = \lim_{(x,y)\to(0,0)} \dfrac{x^4}{2x^4} = \dfrac{1}{2}$

   Hence this limit does not exist.

   (c) $\lim_{(x,y)\to(0,0)} \dfrac{-\sin(x^2)\,y^6}{x^4+3y^4}$

   $0 \le \left| -\sin(x^2)\cdot y^2 \cdot \dfrac{y^4}{x^4+3y^4} \right| \le y^2$ | $\lim_{(x,y)\to(0,0)} y^2 = 0 = \lim_{(x,y)\to(0,0)} -y^2$ (cont.)

   $\Rightarrow -y^2 \le \dfrac{-\sin(x^2)\,y^6}{x^4+3y^4} \le y^2$ | By Squeeze Theorem $\lim_{(x,y)\to(0,0)} \dfrac{-\sin(x^2)\,y^6}{x^4+3y^4} = 0$.

   (d) $\lim_{(x,y)\to(0,0)} \dfrac{x^2ye^y}{x^4+4y^2}$

   $\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \dfrac{x^2ye^y}{x^4+4y^2} = \lim_{y\to 0} \dfrac{0}{4y^2} = 0$ | $\lim_{\substack{(x,y)\to(0,0)\\ y=x^2}} \dfrac{x^2ye^y}{x^4+4y^2} = \lim_{\substack{(x,y)\to(0,0)\\ x=0}} \dfrac{x^4e^{x^2}}{5x^4} = \dfrac{1}{5}$

   Hence $\lim_{(x,y)\to(0,0)} \dfrac{x^2ye^y}{x^4+4y^2}$ does not exist.

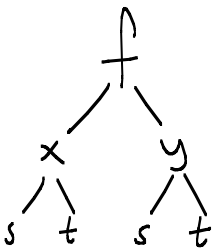
3. (3 +3 = 6 pts.) Find the following partial derivatives for the differentiable function

$$f(x,y) = \int_x^{y^2} \sin(1-t)\,dt \quad = -\int_{y^2}^{x} \sin(1-t)\,dt$$

(a) $\dfrac{\partial f}{\partial x} = -\sin(1-x)\cdot 1$

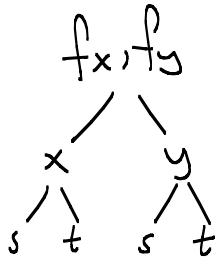
(b) $\dfrac{\partial f}{\partial y} = \sin(1-y^2)\cdot 2y$

4. (4+4+8=16) Find the following partial derivatives for the differentiable function $f(x,y)$ where $x = s^2 + t^2$ and $y = st$.



(a) $\dfrac{\partial f}{\partial s} = f_x \dfrac{\partial x}{\partial s} + f_y \dfrac{\partial y}{\partial s} = f_x \cdot 2s + f_y \cdot t$

(b) $\dfrac{\partial f}{\partial t} = f_x \cdot \dfrac{\partial x}{\partial t} + f_y \cdot \dfrac{\partial y}{\partial t} = f_x \cdot 2t + f_y \cdot s$



(c) $\dfrac{\partial}{\partial t}\dfrac{\partial f}{\partial t} = \dfrac{\partial}{\partial t}\left(f_x \cdot 2t + f_y \cdot s\right)$ (t, t, t)

$= \left[\dfrac{\partial f_x}{\partial t} \cdot 2t + f_x \cdot 2\right] + \dfrac{\partial f_y}{\partial t} \cdot s$

$= \left[\left(f_{xx}\cdot 2t + f_{xy}\cdot s\right)\cdot 2t + f_x \cdot 2\right] + \left(f_{yx}\cdot 2t + f_{yy}\cdot s\right)\cdot s$

5. (6 pts.) Find the equation of the tangent plane to the surface $z = 3^x + e^y$ at the point $(-1,0)$. $z_0 = 3^{-1} + e^0 = \frac{1}{3}$

$$\left.\frac{\partial z}{\partial x}\right|_{(-1,0)} = \left.3^x \cdot \ln(3)\right|_{(-1,0)} = \frac{\ln(3)}{3} \qquad \left.\frac{\partial z}{\partial y}\right|_{(-1,0)} = \left.e^y\right|_{(-1,0)} = 1$$

Tangent plane eqn. : $z = \frac{1}{3} + \frac{\ln 3}{3}(x+1) + 1\cdot(y-0)$

6. (2 pt.) Because of scheduling conflicts, we have to make-up Math 120 on the designated date 21.7.2012. Please answer the following questions with a **Y**(es) or **N**(o).

(a) Are you available on 21.7.2012 between the hours of 09:40 and 12:30? Y

(b) Are you available on 21.7.2012 between the hours of 14:40 and 17:30? Y