

M E T U

Northern Cyprus Campus

Calculus for Functions of Several Variables Midterm II					
Code : <i>Math 120</i>			Last Name:		
Acad. Year: <i>2010-2011</i>			Name :	Student No:	
Semester : <i>Spring</i>			Department:	Section:	
Date : <i>4.30.2011</i>			Signature:		
Time : <i>10:00</i>			6 QUESTIONS ON 6 PAGES TOTAL 100 POINTS		
Duration : <i>120 minutes</i>					
1	2	3	4	5	6

1. (5 pts each) Consider 3 points $P_1(1, 1, -2)$, $P_2(2, 3, -1)$, $P_3(3, 6, 2)$ in the 3-dimensional cartesian space.

(a) Find the linear equation of the plane passing through the points P_1 , P_2 and P_3 .

(b) Find the area of the triangle with vertices P_1 , P_2 and P_3 .

(c) Find the volume of the tetrahedron with vertices O , P_1 , P_2 and P_3 where O is the origin. Hint: The volume of this tetrahedron is equal to $1/6$ times the volume of the parallelepiped determined by the vectors $\overrightarrow{OP_1}$, $\overrightarrow{OP_2}$ and $\overrightarrow{OP_3}$.

2. (5+5 pts) Let $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.

(a) Find the scalar parametric equations of the tangent line to the curve of $\mathbf{r}(t)$ when $t = 1$.

(b) Find the curvature of the curve of $\mathbf{r}(t)$ when $t = 1$.

3. (5 pts eachs) Find the limits below or explain why they do not exist.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin^2 y}{x^2 + y^4}$

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{\tan(x^2 + y^2)}{x^2 + y^2}$

(d) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy - 1}{x^2 + y^2 - 1}$

4. (7+8 pts) Let $f(x, y) = x^2 + y^3$ where $x = s + 2t$ and $y = s^2t$. **USE CHAIN RULE** to find the partial derivatives below. Substitution is to be made only in the last step.

(a) Find $\frac{\partial f}{\partial s}$ in terms of s and t .

(b) Find $\frac{\partial^2 f}{\partial s^2}$ in terms of s and t .

5. (10+10 pts) Let $f(x, y) = \sin\left(\frac{\pi x}{2}\right)e^y$

(a) Find the equation of the tangent plane to the graph of $f(x, y)$ when $x = 1$ and $y = 0$.

(b) Use part (a) to approximate the value of $\sin\left(\frac{0.9\pi}{2}\right)e^{0.1}$.

6. (5+7+8 pts) Let $z = f(x, y) = x^2 + 4y^2$.

(a) Sketch the level curve at $z = 8$.

(b) Find the equation of the normal line at $(2, 1)$ to the level curve in part (a). [The normal line is the line which is orthogonal to the curve at the given point.]

(c) Find the direction \mathbf{u} at which $D_{\mathbf{u}}f(2, 1)$ is maximum.