M E T U

Northern Cyprus Campus

Math 120	**Calculus for functions of several variables**	**I. Exam**	**23.03.2009**
Last Name: Name : Student No:	Dept./Sec. : Time : 17: 40 Duration : 120 *minutes*	Signature	
7 QUESTIONS ON 7 PAGES		TOTAL 100 POINTS	

1	2	3	4	5	6	7			

Q1 (16=4+4+4+4 pts.) Determine whether the following sequences converge or diverge, and find their limits if they converge. Explain your answers.

(a) $a_n = \sqrt{n^2+1} - n$

(b) $b_n = \dfrac{\ln(2n)}{\ln(3n)}$

(c) $c_n = \dfrac{(-1)^n\, n}{n+1}$

(d) $d_n = \dfrac{\sin(\sqrt{n})}{n}$

Q2 (15=5+5+5 pts.) Determine whether the following series converge or diverge. Explain your answers.

(a) $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2+2n}$

(b) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

(c) $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{n}$.

Q3 (14=7+7 pts.) Computing the N-th partial sum $s_n = \sum_{n=1}^{N} a_n$ of the following series, find their exact values $\sum_{n=1}^{\infty} a_n$ if they converge.

(a) $\sum_{n=1}^{\infty} \left(\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{(n+1)\pi}{2}\right) \right)$

(b) $\sum_{n=1}^{\infty} \left(\arctan\left(\frac{n-1}{n}\right) - \arctan\left(\frac{n}{n+1}\right) \right)$

Q4 (15 pts.) Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$ converges or diverges. Check all necessary conditions of the test to be applied.

Q5 (15=3+12 pts.) Consider the series $\sum_{n=0}^{\infty} \frac{(2x+3)^n}{\sqrt{n^2+1}}$.

(a) Write down this series as a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ by indicating the center a and the coefficients c_n.

(b) Find the radius of convergence ρ and the interval of convergence I of this power series.

Q6 (15=8+7 pts.) Consider the function $f(x) = \dfrac{1}{1-x}$ on the interval $-1 < x < 1$.

(a) Using the power series expansion of the function $f(x)$ about $a = 0$ and Term-by-Term differentiation theorem, find the relevant power series expansion of the function $\dfrac{1}{(1-x)^2}$ about the same point $a = 0$.

(b) Find the sum $\sum_{n=0}^{\infty} \dfrac{n}{2^n}$ based upon the result of **(a)**.

Q7 (10=2+3+5 pts.) Consider the function $f(x) = xe^x$.

(a) By induction on n, prove that $f^{(n)}(x) = (x+n)e^x$.

(b) Find the Maclaurin series of $f(x)$ using the result of **(a)** .

(c) Use Taylor's inequality for the remainder $R_n(x)$ of the Maclaurin series of $f(x)$, to show that the Maclaurin series converges to $f(x)$ for all x.