METU - NCC

	Precalculus Midterm	
Code : *Math 100* Acad.Year: *2011-2012* Semester : *Fall* Date : *12.3.2011* Time : *13:40* Duration : *120 minutes*	Last Name: Name : Department: Signature:	Student No.: Section:
	12 QUESTIONS ON 4 PAGES TOTAL 100 POINTS	

1 (8)	2 (7)	3 (8)	4 (7)	5 (14)	6 (7)	7 (7)	8 (7)	9 (7)	10 (14)	11 (7)	12 (7)	

1. *(8 pts)* Find the set of solutions to the inequality $|2x-1| > x+5$.

For $x \geq \frac{1}{2}$

$2x-1 > x+5 \Rightarrow x > 6$

For $x < \frac{1}{2}$

$-2x+1 > x+5$

$-4 > 3x$

$x \in (-\infty, -4/3) \cup (6, \infty)$

2. *(7 pts)* The sum of the ages of three brothers is equal to 29. Also, the sum of the ages of the younger two brothers is 5 more than the age of the oldest. Find the age of the oldest brother.

$a+b+c = 29 \quad a \leq b < c$

$a+b = c+5$

$2c+5 = 29$

$c = 12$

12

3. *(8 pts)* Find the set of solutions to the inequality $x^2+8x+11 < 2x+3$.

$x^2+6x+8 < 0$

$(x+2)(x+4) < 0$

		-4		-2	
x^2+6x+8	$+$	0	$-$	0	$+$

$x \in (-4, -2)$

4. *(7 pts)* Find real numbers a and b so that $a+bi = \frac{3-i}{2+i}$.

$\frac{3-i}{2+i} \cdot \frac{(2-i)}{(2-i)} = \frac{6-5i+i^2}{4-i^2} = \frac{5-5i}{5}$

$a = 1 \qquad b = -1$

5. *(2x7 pts)* Let $f(x) = x^2 + 3x + 2$.

a) Complete $f(x)$ to a square. [Find h, k such that $f(x) = (x+h)^2 + k$]

$f(x) = x^2 + 3x + \frac{9}{4} + 2 - \frac{9}{4}$

$f(x) = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$

b) Find the vertex of $f(x)$.

vertex $= \left(-\frac{3}{2}, -\frac{1}{4}\right)$

6. *(7 pts)* Find the coordinates of a point $\mathbf{P} = (x, y)$ so that the distance from $(-7, 9)$ to $\mathbf{P}$ is one third of the distance from $(5, -7)$ to $\mathbf{P}$.

$\frac{5-(-7)}{4} = \frac{12}{4} = 3$

$\frac{|-7-9|}{4} = 4$

$\mathbf{P} = (-4, 5)$

$x = -7 + 3$

$y = 9 - 4$

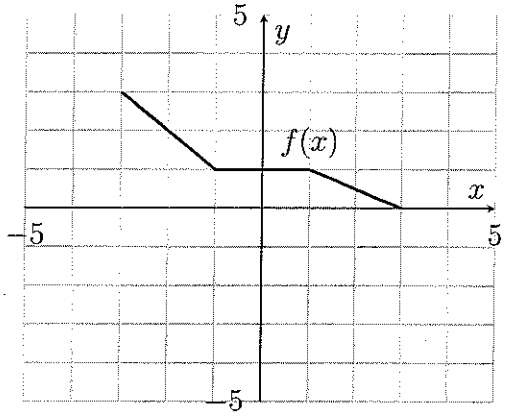
7. *(7 pts)* Find an equation of the line passing through the points $(-3, 4)$ and $(1, -8)$. (Write your answer in standard form: $y = mx + b$)

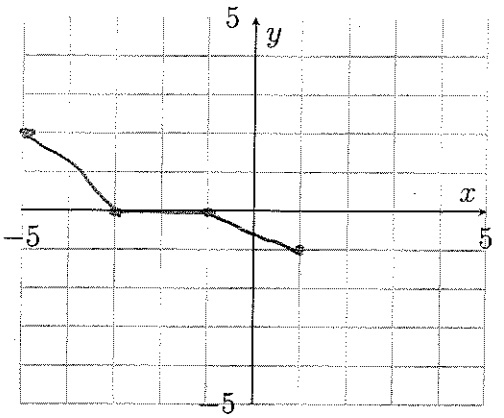
$\frac{x+3}{y-4} = \frac{1+3}{-8-4} = -\frac{1}{3}$

$y = -3x - 5$

$3x + 9 = -y + 4$

8. *(7 pts)* The graph of $f(x)$ is given below. Graph $g(x) = f(x+2) - 1$ in the blank grid provided.





9. *(7 pts)* Describe a function $g(x)$ in terms of $f(x)$ if the graph of g is obtained by shifting the graph of f to the left by 5 units and down by 4 units.

$g(x) = f(x+5) - 4$

10. *(2x7 pts)* Complete the table below by filling in correct values of $(f \circ g)(x)$ and $(f \circ g^{-1})(x)$:

x	1	2	3	4
$f(x)$	4	1	2	3
$g(x)$	2	4	1	3
$(f \circ g)(x)$	1	3	4	2
$(f \circ g^{-1})(x)$	2	4	3	1

11. *(7 pts)* Divide $P(x) = x^5 - 3x^3 + 2x + 4$ by $Q(x) = x^2 - 4$.

$$\frac{P(x)}{Q(x)} = x^3 + x + \frac{6x+4}{x^2-4}$$

$$\begin{array}{r|l} x^5 - 3x^3 + 2x + 4 & x^2 - 4 \\ \cline{2-2} x^5 - 4x^3 & x^3 + x \\ \cline{1-1} \quad x^3 + 2x + 4 & \\ \quad x^3 - 4x & \\ \cline{1-1} \quad\quad 6x + 4 & \end{array}$$

12. *(7 pts)* Use the remainder theorem to find $f(7)$ where $f(x) = x^5 - 6x^4 - 13x^3 + 21x + 4$.

$f(7) = -1907$

$$\begin{array}{r|l} x^5 - 6x^4 - 13x^3 + 21x + 4 & x - 7 \\ \cline{2-2} x^5 - 7x^4 & x^4 + x^3 - 6x^2 - 42x - 273 \\ \cline{1-1} \quad x^4 - 13x^3 + 21x + 4 & \\ \quad x^4 - 7x^3 & \\ \cline{1-1} \quad\quad -6x^3 + 21x + 4 & \\ \quad\quad -6x^3 + 42x^2 & \\ \cline{1-1} \quad\quad -42x^2 + 21x + 4 & \\ \quad\quad -42x^2 + 294x & \\ \cline{1-1} \quad\quad\quad -273x + 4 & \\ \quad\quad\quad -273x + 1911 & \\ \cline{1-1} \quad\quad\quad\quad -1907 & \end{array}$$